Free Slip Plane Analysis of a Strip Footing using a Genetic Algorithm

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ABSTRACT: A strip footing is a uniform load with infinite length in the third dimension. The safety of such a load has been subject of investigation for a quite a long time. By ignoring the weight and internal friction of the soil, a number of upper- and lower bounds can be derived for the bearing capacity. This paper presents an upper bound method that gives the same result as the lower bound. In other words, the exact bearing capacity is derived. Even though the algorithm used is presented on a basic case, any geometry and subsoil configuration can be analyzed.

1 INTRODUCTION

A relatively simple problem in geotechnical engineering is the design of an infinitely long strip footing on a layer of homogeneous cohesive material. Such a strip footing is presented schematically in figure 1 to the right. Because of its relative simplicity, especially when the weight of the material is disregarded, it is possible to determine upper and lower limits for the stability of the footing.

Lower bounds of this equilibrium system can be derived using an equilibrium system. This system consists of a field of stresses that satisfy the boundary conditions and adheres to the yield conditions in any point. A more elaborate discussion on lower bound approaches can be found in (Verruijt, 2007)

Upper bounds for the failure load can be obtained by defining a slip surface and determining the driving and resisting forces along that surface. This displacement field can, for example, have the shape of half a circle. Bishop's method can, in the case of a circular slip plane, give a value for an upper bound of the failure load.

Prandtl's solution to determine of the failure load of a half plane carrying a strip load is the most famous one with the lowest bearing capacity: $c \times (\pi + 2)$. An elaborate deviation of upper bound approaches such as Prandtl can also be found in (Verruijt, 2007)

In the recent past, very effective search mechanisms are developed to find the stability factor using limit equilibrium methods on a free slip plane. This paper shows the representative slip plane with Spencer's limit equilibrium method (Spencer 1967) for a strip footing. Spencer's method can be seen as an upper bound just like Bishop's method (Bishop, 1955).

2 LOWER BOUND OF BEARING CAPACITY

The lower bound of the equilibrium system can best be illustrated using Mohr's circles. In order that all circles remain within the yield envelope, figures 1 and 2 show that the value of the load P must be such that P < 4c.

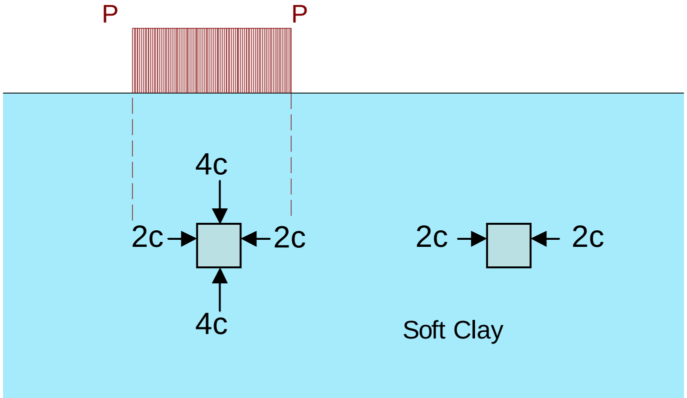

Figure 1: stresses underneath a strip footing

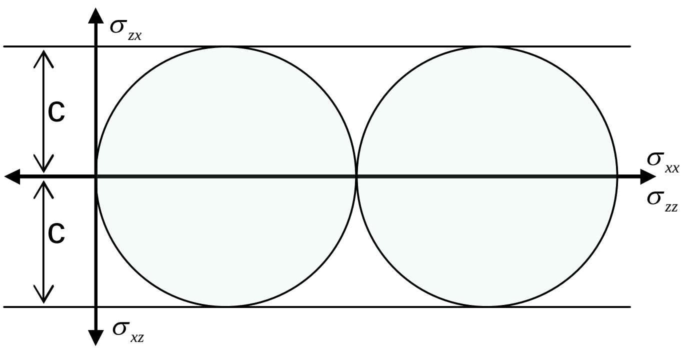

Figure 2: Mohr's circles to determine stresses under the footing

3 UPPER BOUND OF BEARING CAPACITY

An upper bound for the failure load of the strip footing can be obtained by considering a predefined mechanism. Figure 3 shows a circular slip plane with the centre point on the edge of the footing and the diameter equal to the width of the footing. The bearing capacity can be calculated analytically using the virtual work principle. A rotation over a small angle α causes a displacement of α times the radius, 'r'. Assuming that the shear stresses reach their full capacity by this small displacement, the work done by the stresses equals $\pi c \alpha r^2$. The average displacement of the strip footing equals $0.5 \times \alpha r$ therefore, the work performed by the load equals $0.5 \times p \alpha r^2$. Setting the work performed by the stresses equal to the work performed by the load, it follows that $p = 2\pi c = 6.28c$.

This same upper bound can be found with Bishop's method. By setting the cohesion equal to the load, the result of the Bishop analysis of the calculation in figure 3 gives a safety factor of exactly 6.28.

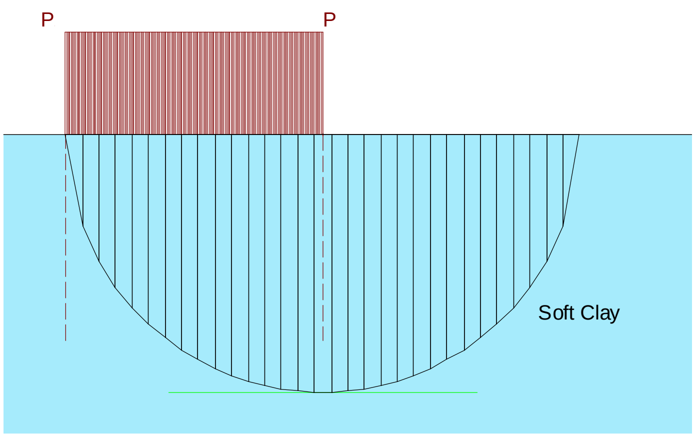

Figure 3: Upper bound by Bishop's method, f=6.28

The slip circle in figure 3 is an arbitrary one. By having a higher center point and decreasing the angle of the circle from 180 degrees to 134 degrees, one finds a maximum bearing capacity of $p = 5.52c$

Once again, exactly the same value can be obtained with Bishop's method. By defining a fine grid and a significant number of tangent lines, the representative slip circle can be calculated. Figure 4 shows the geometry used for a Bishop analysis to calculate the stability of the load. The safety factor of this analysis, using c=p exactly gives a safety factor of 5,52.

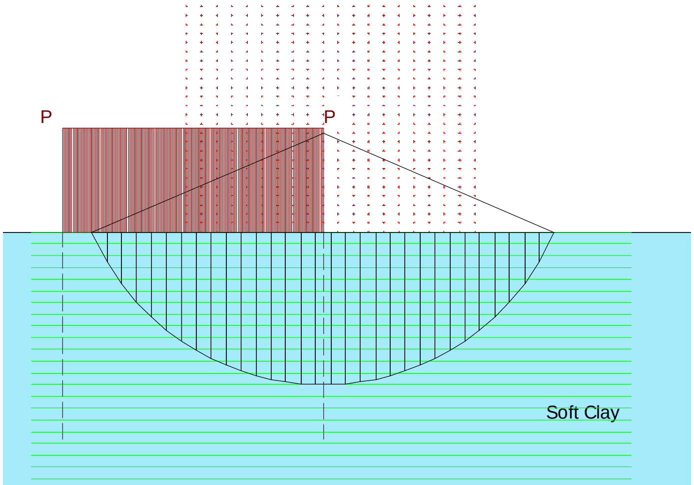

Figure 4: Upper bound by Bishop's method, f=5.52

The most renowned derivation of the failure load is the one by Prandtl. Prandtl divides in figure 5 the underground into three area's: two triangular wedges (area's 1 and 3) where in between a quarter of a circle is placed (area 2).

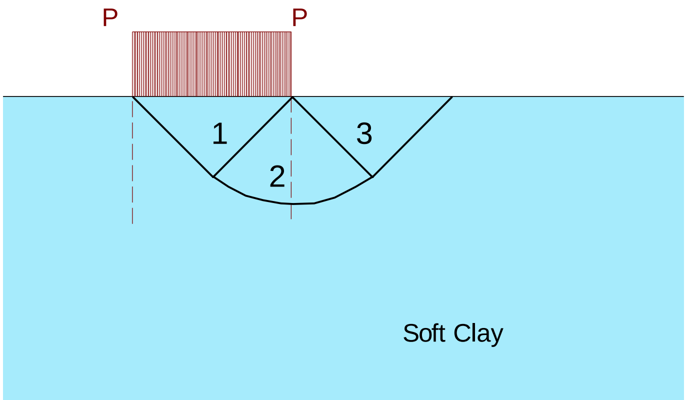

Figure 5: Prandtl's method

The easiest way to determine the maximum bearing capacity according to Prandtl is to consider the equilibrium of each area. Doing so results in a bearing capacity of $p = (\pi + 2)c = 5{,}14 \times c$.

Bishop's method cannot determine the safety along such a slip plane. Instead, Spencer's method can be used with the slip plane from figure 5. The safety factor with Spencer's method that belongs to the slip plane (figure 6) is 5.14.

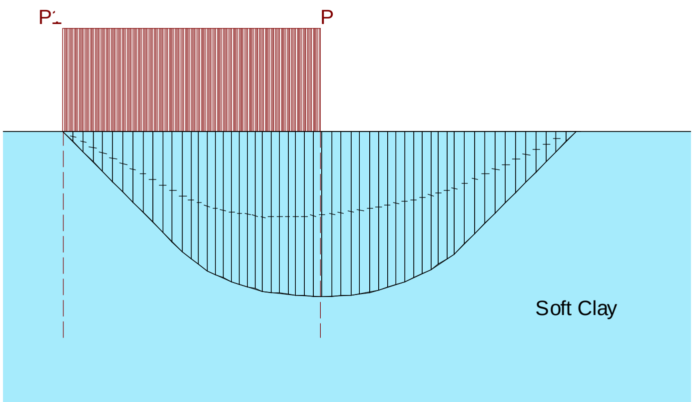

Figure 6: Upper bound by Prandtl, f=5,14

Currently, we are at a similar point as in figure 3. An upper bound is found, but one cannot be sure that this is the lowest one. Figure 4 improves the slip plane of figure 3 by an optimization method. The shapes of area's 1, 2 and 3 in Prandtl's method can also be optimized in order to find a lower safety factor. The following sections of this paper concern how the representative slip plane will be found with a lower upper bound than Prandtl's solution.

4 GENETIC ALGORITHMS

Several computational programs are available, with which the stability of a soil body can be calculated with a limit equilibrium method. As shown in Figure 7, the user enters an area in which the program needs to find the slip plane with the minimal stability factor.

Searching such a space usually happens by calculating all possible slip circles with corresponding tangent lines and reporting the one with the minimal safety. This algorithm is very time consuming and does not guarantee a global minimum.

Other search routines (for example hill climbing) have great disadvantages as well. In the recent past Genetic Algorithms *(Barricelli, Nils Aall 1957)* are used more frequently as a search procedure and it seems to be a well-suited method to find the representative slip plane with the minimal safety factor.

Genetic algorithms process a mathematical representation of a solution of an analyzed problem. For Bishop's method, this representation is a vector containing the X and Y value of the centre of the circle, and the radius of the circle. This representation can be seen as an individual. The group of individuals forms a population. An individual can be tested for its fitness, for example with Bishop's method.

The genetic algorithm improves the quality of a population in a similar way as nature. Two individuals cross their DNA, there is a chance for mutations and a new individual is created. Two new individuals fight, and the fittest one survives to the next generation.

The algorithm seems to be faster and better at finding a global minimum. A disadvantage is that the results are not always reproducible. On top of that, there will be a very strong tendency to find the global minimum, while sometimes, a local minimum is interesting as well. This can be overcome using penalties steering the result in the desired direction. Because of its high speed, a genetic algorithm makes it possible to find a free slip surface with Janbu's or Spencer's method. (Van der Meij and Sellmeijer, 2010)

Such a free slip plane should have at least 10 degrees of freedom in order to be able to find any slip plane. With a grid based method, it is too time consuming to search a 10 dimensional, often very complex search area.

The approach by which the genetic algorithm finds the free slip plane is also drawn in figure 7. An upper- and lower bound is defined with the same number of point. In figure 7, the number of points along a plane is 13. The upper and lower bounds are connected with straight lines. The first and last ones are connected over the surface line. The free slip plane consists of points along these transversal lines. In this figure, a completely random slip plane is drawn between the upper and lower bounds. The genetic algorithm is able to move the points along the transversal line in order to minimize the safety factor. The calculation with the lowest safety factor is the representative slip plane with its associated safety.

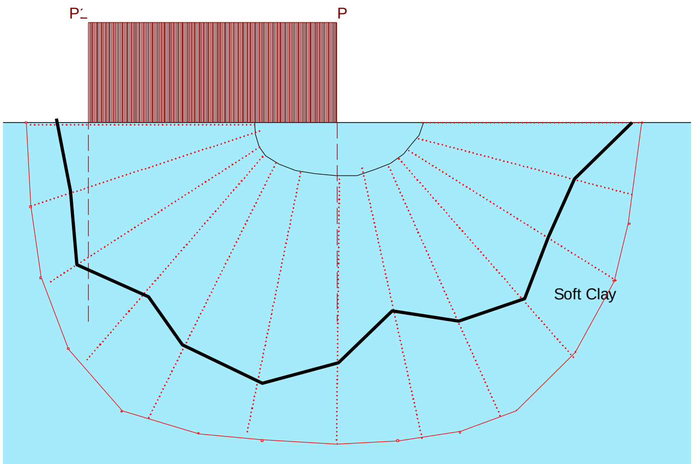

Figure 7: free definition of Spencer's slip plane

5 SEARCH FOR THE LOWEST UPPER BOUND

Spencer's limit equilibrium method converges to the slip plane given in figure 8. The safety factor that belongs to this mechanism under the assumption of c = p is exactly 4.00

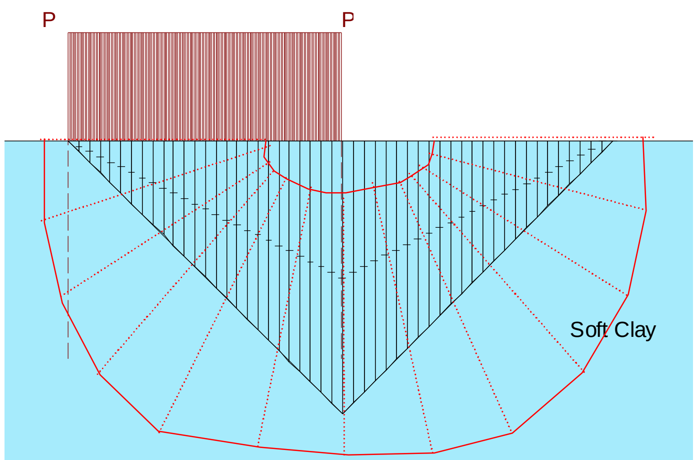

Figure 8: Upper bound by genetic algorithm with Spencer's method, f=4,00

This upper bound safety factor is identical to the lower bound presented in the beginning of this paper.

In essence, we are back to figure 1 of this paper. The stresses underneath the load strip are drawn in figure 9 and – even though they are drawn on a different plane – they represent the same stress state as in the first figure.

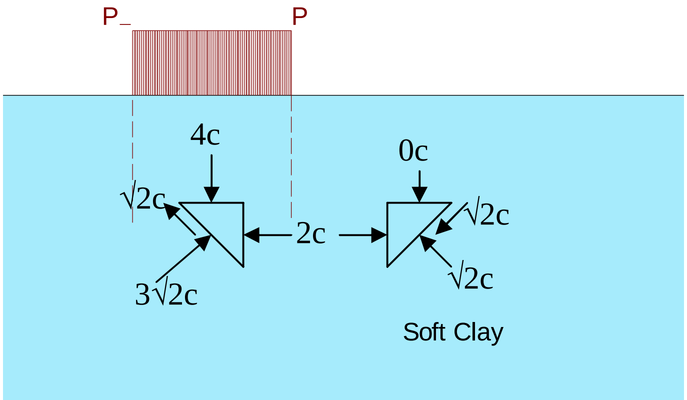

Figure 9: stresses underneath the strip load from figure 9

6 CONCLUSIONS

This exercise proves that Spencer is a good limit equilibrium method as it gives the lowest possible value for the safety factor. In combination with a genetic algorithm, it can perform a truly free search. It does not only find a global minimum safety factor of this simple strip footing, but for any load or embankment on any subsoil. This makes its applicability very broad.

One cannot prove that Spencer's method in combination with this search algorithm will always result in a safety factor that is equal to a theoretical lower bound, but this paper does show that this is a better approach than Bishop's method with circular slip planes or Prandtl's method.

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