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NONLINEAR MODEL PREDICTIVE CONTROL OF FLOOD DETENTION BASINS IN OPERATIONAL FLOOD FORECASTING

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We present a Nonlinear Model Predictive Control (NMPC) algorithm for the operational control of flood detention basins. The algorithm consists of an iterative, finite horizon optimization of a water resources system over a short-term control horizon. The underlying nonlinear model simulates the hydraulics of the water resources system by a kinematic wave approach including additional formulas for the hydraulic structures. Objectives of the control, i.e. the desired damping of flood peaks, and constraints such as the water level dependent capacity of the structures are mathematically formulated in a set of objective functions and inequality constraints. The resulting optimization problem is solved by Sequential Quadratic Programming (SQP). We present an application of the algorithms to the integral control of six hydraulic structures and two major flood detention basins along the bifurcation points of the Rhine River in the Netherlands.

INTRODUCTION

The most common technique for supervisory control of hydraulic structures in water resources is the definition of explicit operating rules. Examples include minimum releases for reservoirs due to the reservoir level and environmental objectives, the operation of flood detention basins based on water level at reference locations, or the definition of set points for upstream water levels of river weirs. These operating rules typically come along with secondary controllers for controlling the desired variable at site, i.e. a PID-controller for maintaining an upstream water level at a weir.

Whereas this concept works well for smaller water systems, its application gets significantly more complex for larger systems, in particular if these systems have a high degree of interconnectivity such as the Dutch Rhine-Meuse delta. In these cases, the operating rules and the water system may show undesired feedback effects leading to suboptimal control of the total system. Looking for example at the control of cascaded hydropower plants, many authors such as Ackermann et al. [1], Glanzmann et al. [3], or

Pfuetzenreuter & Rauschenbach [5] report drawbacks of classical feed forward / feedback control methods due to amplification of inflow disturbances. In recent yars, the solution to this problem is found in the application of Model Predictive Control (MPC) for supervisory control of weirs [4] and hydropower plant.

MPC is a control concept, which has become an industrial standard in process control over the last two or three decades. It makes use of a process model for predicting future trajectories of the controlled variables over a finite horizon in order to determine the optimal set of manipulated variables by an optimization algorithm. An integral part of the concept is the explicit consideration of constraints on inputs, states and outputs. Furthermore, the tuning of the control parameters is not very tedious even in the presence of contradictory control objectives.

In the next section, we present a general nonlinear MPC scheme for the control of hydraulic structures in complex river networks. The underlying process model, described in the following section, is a hydraulic model based on the kinematic wave assumptions. Finally, we present the operational application of the NMPC on the control of several hydraulic structures of two flood detention basins and the discharge distribution at the bifurcation points of the River Rhine in The Netherlands.

NONLINEAR MODEL PREDICTIVE CONTROL SCHEME

Under the assumption that we already applied a spatial schematization to our system of interest, the model of a water resources system can be described by the following generic set of non-linear ordinary differential equation (ODE):

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = f(\mathbf{x}, \mathbf{u}, \mathbf{d}) \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^l$ is the system state vector, $\mathbf{u} \in \mathbb{R}^m$ the vector of controlled variables, $\mathbf{d} \in \mathbb{R}^n$ the vector of disturbances, l the number of states, m the number of controlled variables, and n the number of disturbances. A formulation of the MPC on-line optimization is given by the solution of the following optimum control problem over a finite prediction horizon $t \in \{0..T\}$ by

$$\min_{\mathbf{u} \in \{0..T\}} J(\mathbf{x}, \mathbf{u}) \tag{2}$$

subject to the system dynamics (1) and p additional inequality constraints

$$g_i(\mathbf{x}, \mathbf{u}) \leq 0, \quad i \in I = \{1, \ldots, p\} \tag{3}$$

We transform the set of ODE (1) into a discrete-time system under the assumption of an explicit time stepping scheme and get

$$\mathbf{x}^{k+1} = f(\mathbf{x}^k, \mathbf{u}^k, \mathbf{d}^k) \tag{4}$$

where k is the time step index. The solution of equation (4), i.e. the computation of new states $\mathbf{x}^{k+1}$ based on the data of a previous time step $\mathbf{x}^k$, $\mathbf{u}^k$ and $\mathbf{d}^k$, is performed over the finite prediction horizon under consideration of an initial condition $\mathbf{x}^0$.

The resulting optimum control problem (2-4) is solved by the nonlinear programming scheme SNOPT [2]. It becomes available in the commercial optimization toolbox TOMLAB (http://tomopt.com/tomlab/) under Matlab.

INTERNAL MODEL

The open-channel flow in a river network in one dimension is described by the De Saint-Venant equations consisting of mass (continuity) and momentum conservation. The continuity equation reads:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_{lat} \tag{5}$$

while the non-conservative form of the momentum equation is defined by:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial h}{\partial x} = -cf\frac{v|v|}{m}, \quad cf = \frac{g}{C^2} \tag{6}$$

where A = wetted area, Q = discharge, q_{lat} = lateral discharge per unit length, h = water level, v = flow velocity, g = acceleration due to gravity, m = hydraulic radius, C = Chezy coefficient, cf = dimensionless bottom friction coefficient.

The kinematic wave equations can be derived from the complete hydraulic model (5, 6) by neglecting of the terms for inertia and convection in the momentum equation (6). By additional substitution of $v = Q/A$ equation (2) becomes

$$g\frac{\partial h}{\partial x} = -\frac{gQ|Q|}{C^2 A^2 m} \tag{7}$$

We now apply a spatial schematisation on a staggered grid, on which the discharge is schematised between an upstream and a downstream storage node including a discrete water level. Defining the distance of these nodes to be Δx equation (7) can be rearranged to

$$Q = f_{flow}(h_{up}, h_{down}) = -\text{sign}(h_{up} - h_{down})CA\sqrt{\left|\frac{h_{up} - h_{down}}{\Delta x}\right| m} \tag{8}$$

where *C*, *A*, *m* can be expressed as functions of the mean water level $(h_{up} + h_{down})/2$. If hydraulic structures exist between two storage nodes, the flow equation (8) can be replaced by a general equation of the hydraulic structure, given by

$$Q = f_{structure}(h_{up}, h_{down}, dg) \tag{9}$$

where dg = gate or weir setting.

The numerical solution of the continuity equation (5) is done by the Euler Forward Method resulting in:

$$\frac{A(h^k) - A(h^{k-1})}{\Delta t} + \frac{Q_{down}^{k-1} - Q_{up}^{k-1}}{\Delta x} = q_{lat}^{k-1} \tag{10}$$

in which *k* denotes the time step. By substituting $s(h) = A(h)\Delta x$, multiplying Δx, neglecting the lateral discharge, and introducing equations (8, 9) by replacing the water level *h* by the storage *s*, we may transform equation (10) into a water balance at node level

$$s^k = s^{k-1} + \Delta t \sum_i f(s^{k-1}, s_i^{k-1}, dg_i^{k-1}) \tag{11}$$

where s = storage at a node, i = the index of connected branches to the storage node.

We set-up an adjoint system for computing the gradient of *J* related to the controlled variables *dg*. The Lagrangian form of the optimum control problem over the prediction horizon reads

$$L = J(\{s^k, s_i^k, dg_i^k\}_{k=1}^T) + \sum_{k=1}^{T} \lambda^k \{s^k - s^{k-1} - \Delta t \sum_i f(s^{k-1}, s_i^{k-1}, dg_i^{k-1})\} \tag{12}$$

in which λ = Lagrange multiplier. In this formulation, we define the system equations at each discrete time step as separate equality constraints of the optimum control problem. We now apply a variational analysis of the Lagrangian form, sort all terms according to spatial derivatives and receive

$$\lambda^{k-1} = \lambda^k - \frac{\partial J}{\partial s^{k-1}} - \Delta t(\lambda^k - \lambda_i^k) \sum_i \frac{\partial f(s^{k-1}, s_i^{k-1}, dg_i^{k-1})}{\partial s^{k-1}} \tag{13}$$

$$\frac{dJ}{d(dg^{k-1})} = \frac{\partial J}{\partial dg^{k-1}} + \Delta t(\lambda_{up}^k - \lambda_{down}^k) \frac{\partial f(s_{up}^{k-1}, s_{down}^{k-1}, dg^{k-1})}{\partial dg^{k-1}} \tag{14}$$

The procedure of computing the objective function value and its gradient can be summarised as follows:

1. A model simulation is performed forward in time by applying equation (11). The objective function value J is computed.
2. The Lagrangian multiplier λ^k is computed backwards in time by applying equation (13).
3. The gradient of $dJ/d(dg^k)$ is computed by equation (14).

The procedure above computes a gradient of the cost function J related to a set of m controlled variables by computational costs comparable to a model simulation itself. The numerical alternative would require m executions of the simulation model and therefore requires significantly more computational resources.

BIFURCATION TEST CASE

The bifurcation points of the River Rhine are the key to the discharge distribution along the different Dutch river Rhine branches and therefore have a major impact on the water management in The Netherlands. The discharge distribution affects various aspects such as the allocation of drinking water, irrigation, salt intrusion, navigation, and flood protection. The control of the discharge distribution has been the focus of several recent publications such as Schielen et al. [6].

In this paper, we present the set-up of an NMPC implemented as a pilot in the Dutch flood forecasting system for the rivers Rhine and Meuse. It controls the discharge distribution at the bifurcation points (Figure 1) at low and medium flows by control of a hydraulic structure at Driel (S01). It operates five inlet and outlet structures (S02-S06) of two projected flood detention basins for dampening flood peaks during flood events.

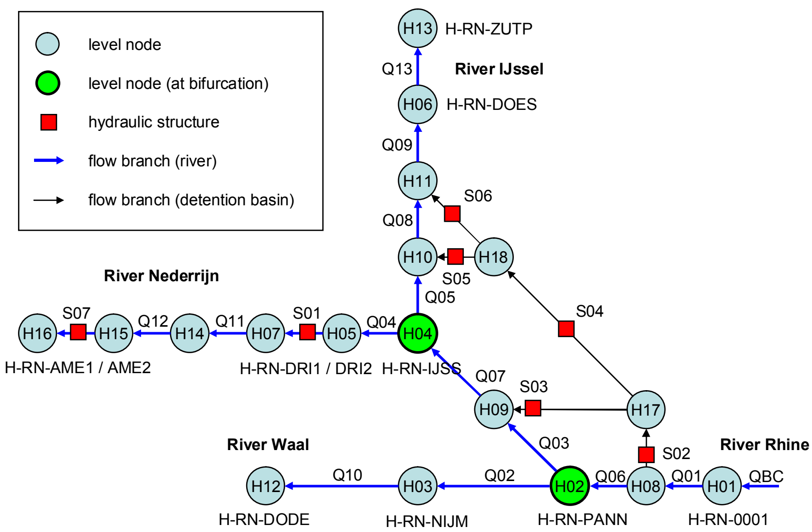

Figure 1 Layout of kinematic wave model: schematic overview about nodes and flow branches and hydraulic structure branches

The model topology of the kinematic wave model was set-up as course as possible in order to reduce CPU time. Nodes are only placed at gauges, bifurcation points, upstream and downstream of hydraulic structures in the rivers and inlet / outlet structures of the detention basins. One additional calculation point had to be placed in one of the largest branches to increase the model accuracy.

We select the period from October 2007 until December 2008 for model calibration. The calibration is performed manually by adapting the level-dependent roughness of each flow branch until the Root Mean Square Error (RMSE) has been observed in the order of 10 cm for the difference of hourly observed and simulated water level at selected gauges. The complete set of performance indicators such as Bias, Root Mean Square Error (RMSE), Nash-Sutcliffe Efficiency Index for the calibration is presented in Table 1.

Table 1 Performance indicators for model calibration, period October 2007 until December 2008

Location	Bias [cm]	RMSE [cm]	Nash-Sutcliffe [-]
Lobith	-5.4	9.2	0.99
Pannerdenschekop	-7.0	10.7	0.99
Nijmegenhaven	-0.9	10.9	0.99
IJsselkop	-5.5	10.3	0.98

The control horizon of the NMPC is 5 days with time steps of 2 minutes resulting in a total number of 3600 time steps. We choose a control time step of one hour, i.e. 120 control time steps for each hydraulic structure.

The objective function value $J(\mathbf{x},\mathbf{u})$ is defined as

$$\begin{aligned} J = & \sum_{k^*,k^{**},i} w_{dg,i}(dg_i^{k,k})^2 + \sum_{k^*,k^{**},i} w_{\Delta dg,i}(dg_i^k - dg_i^{k-1})^2 \\ & + \sum_{k^*,i=1} w_{h,i}(h_i^k - h_{setpoint})^2 + \sum_{k^{**},i=2..6} w_{h,i}[\max(h_i^k - h_{threshold},0)]^2 \end{aligned} \tag{14}$$

in which $w_{dg,i}$, $w_{\Delta dg,i}$ and $w_{h,i}$ are the weighting coefficients for penalizing the use of hydraulic structures, the deviation from water level set points $h_{setpoint}$, and the crossing of maximum allowed water levels $h_{threshold}$. Furthermore, we limit the evaluation of some terms to specific flow conditions, e.g. the term of the water level set point is only computed at time steps k^* corresponding to low flow and medium flow regimes for which the set point can be achieved by the control of structure S01. In other cases, i.e. flow events, the gate of the structure is fully opened. The most downstream hydraulic structure at Amerongen (S07) does not have any impact on the optimum control problem. Therefore, we set its gate setting dg_7 to a constant value.

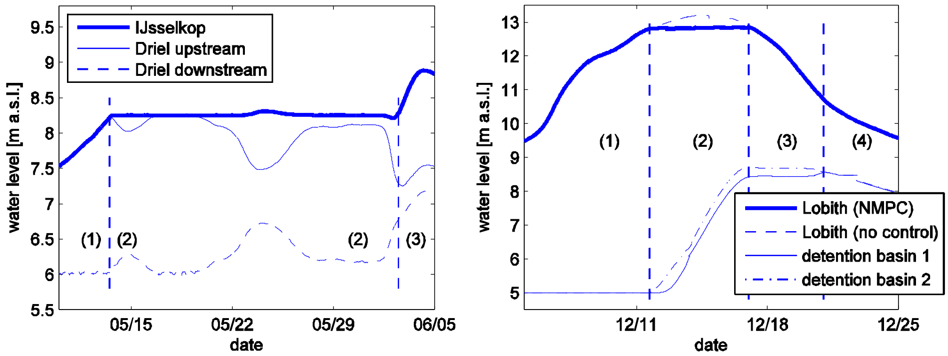

Figure 2 a) water level control at Driel during low - medium flow regime in May 2007 with water level set point of 8.25 m a.s.l. at gauge IJsselkop, b) damping of small flood peak above 12.75 m a.s.l. in December 2007 at gauge Lobith by control of detention basins 1 and 2

Figure 2 presents some results of the NMPC running in a closed loop setting using the kinematic wave model also as a replacement of the actual system and perfect predictions of the disturbance. We intend to repeat the exercise in the near future using a full hydraulic model and predicted disturbance.

In the left figure, the regime is gradually shifting from low flow (1) for which the set point is not maintained even with fully closed gates, to (2) medium flow for which the set point is well maintained, to (3) a higher flow regime with gates completely opened and balanced water levels upstream and downstream of the gate. The right figure presents the dampening of a small flood wave. In phase (1) the inlet structures of the detention basins are still inactive. They start discharging the water during phase (2) for keeping the water level at Lobith at a level of 12.75 m a.s.l. Inlet gates are closed again in phase (3) till the water is released from the detention basins through the outlet structures in phase (4).

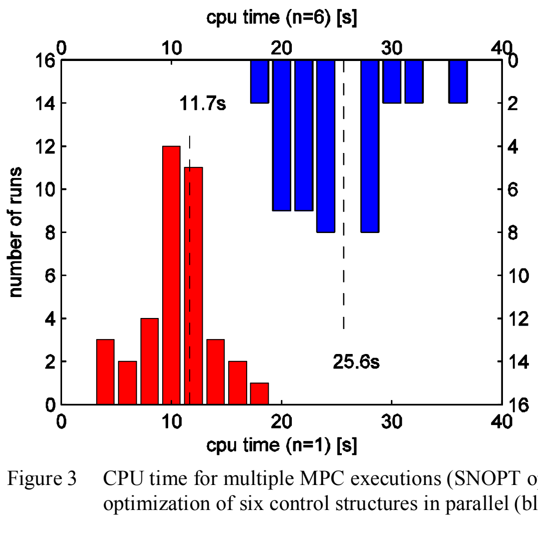

Table 2 CPU profile of single MPC application (SNOPT optimizer) with parallel optimization of six hydraulic structures

	CPU time [s]	CPU time [%]
optimizer	0.221	0.94
internal model	23.101	97.93
Matlab overhead	0.267	1.13
	23.589	100.00

Figure 3 CPU time for multiple MPC executions (SNOPT optimizer), single control structure (red) and optimization of six control structures in parallel (blue)

The application of the NMPC within the operational flood forecasting system limits its acceptable run time to a desired maximum of about one minute. Figure 3 shows that this goal was achieved for both optimization problems with one structure (120 dimensions) and six structures (720 dimensions). Note that the execution time of the much larger problem is only 2.5 times that of the small one. Profiling of the application (Table 2) shows that the reason for this is the high proportion of model execution related to the total CPU time. In our opinion, this shows quite a lot of potential to even speed up the NMPC by implementing for example an implicit time stepping.

CONCLUSIONS

This paper presents a Nonlinear Model Predictive Control scheme for supporting operational decision-making on hydraulic structures. It is applied on the control of flood detention basins in a complex river network. Since the proposed framework is generic and allows for the straightforward integration of arbitrary process models and control objectives, it will be applicable to various other applications in water resources such as the control of cascades of hydropower plants or irrigation systems.

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