Multiple equilibria for suspended sediment concentrations in rivers and tidal flows

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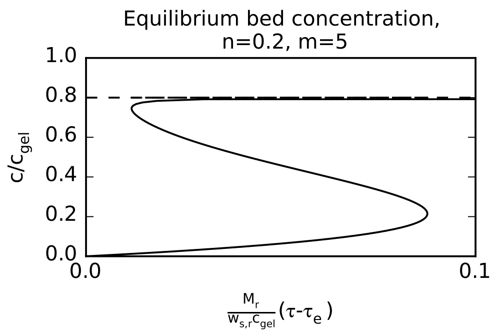
Abstract

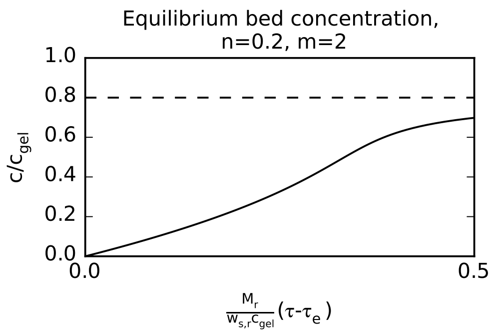
Some estuaries have relatively low suspended sediment concentrations of the order of 100 mg/l, while others are hyper-turbid, with sediment concentrations up to several tens of grams per litre. Over the last decades we also have seen estuaries, such as the Ems and Loire, where a transition occurred from a low to a high concentration, possibly due to human interventions in the estuarine system. Such a *regime shift* can potentially have devastating effects on the quality of the ecosystem. Winterwerp & Wang (2013) hypothesise that the low and high concentration conditions may both be equilibrium states of an estuary under the same forcing conditions. In this presentation we will test this hypothesis and investigate the behaviour of the most essential physical processes. This is done using a numerical water column model.

It is found that the formulation of deposition and erosion of sediment at the bed is of crucial importance to the existence of multiple equilibria. The deposition $w_s c$ is affected by hindered settling, which is modelled as $w_s = w_{s,r}(1-\phi)^m$ (Richardson & Zaki, 1954), where $\phi = c/c_{\mathrm{gel}}$, c is the concentration, c_{gel} is the gelling concentration and m is a parameter determined from measurements. The erosion $M(\tau - \tau_e)$ (for $\tau > \tau_e$) depends not only on the bed shear stress τ and critical shear stress τ_e, but is also affected by hindered erosion. This means the erosion rate is reduced if the flow is already carrying a large sediment load. This effect is modelled by reducing the erosion factor M according to $M = M_r(1-n-\phi)/(1-n)$, where n is the porosity and M_r is a reference erosion coefficient.

The balance between erosion and deposition at the bed determines the equilibrium solutions of the water column model. Fig 2 shows possible near-bed equilibrium concentrations as a function of a dimensionless shear stress for two different choices of the hindered settling parameter m and the porosity n. For parameter values $m = 5$ and $n = 0.2$ (Fig 2a) it is found that there is a range of shear stresses with three possible equilibrium concentrations at low, high and hyper-concentrated levels. For other values $m = 2$ and $n = 0.2$ (Fig 2b) there is just one single equilibrium for each value of the shear stress and small changes in the shear stress do not lead to sudden large changes in the concentration. The exact parametrisation of the boundary condition is thus essential to the existence of multiple equilibria.



(a)



(b)

Figure 1: Equilibrium concentrations at the bed as a function of a dimensionless shear stress following from the Partheniades-Krone boundary condition with hindered settling and hindered erosion. Two different settings for the hindered settling exponent m are plotted assuming a porosity $n = 0.2$. For $m = 5$ there is a range of shear stresses where there are three equilibria, while for $m = 2$ only one equilibrium state is possible for each shear stress.

For a steady uniform river flow, this bed shear stress is a complex function of the discharge velocity, that depends strongly on the damping of turbulent mixing by sediment stratification. To find this relation, we use erosion/deposition parameters $m = 5$ and $n = 0.2$ and compute the turbulent mixing using the $k-\varepsilon$ turbulence closure. Using these results,

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Fig 2 again shows the near-bed equilibrium concentration, but now as a function of the depth-averaged velocity. We still see multiple equilibria if the erosion parameter M_r is sufficiently small (blue line), but the multiple equilibria disappear when the erosion parameter is increased (green line). The higher erosion parameter means more erosion of sediment and therefore stronger sediment stratification. This stratification reduces turbulent mixing and suppresses the bed shear stress, especially at high sediment concentrations. Situations with small velocities then no longer produce sufficient shear stress to support high concentrations, so rendering the combination small velocity - high concentration infeasible. So, for river flows, the existence of multiple equilibria as a function of the discharge velocity depends strongly on the erodability of sediments at the bed.

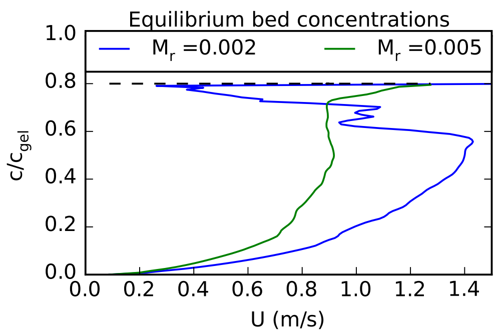

Figure 2: Concentration at the bed as a function of the depth-averaged velocity U in a stationary uniform flow. If the erosion coefficient is sufficiently small (blue line), there is still a range of velocities with multiple equilibrium solutions. The multiple equilibria disappear for higher erosion coefficients (green line).

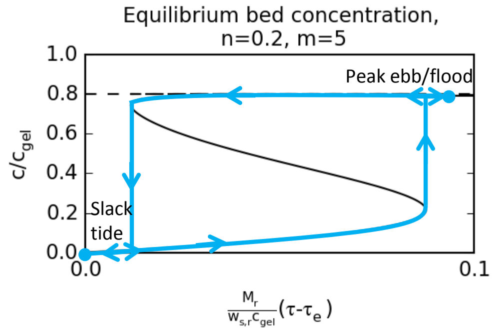

Figure 3: Schematic representation of the bed concentration during a strong slow tidal flow. The concentration is relatively low between slack tide and peak tide, then jumps up the before peak tide. The concentration then remains high between peak tide and slack tide, before it jumps back to low concentrations just before slack tide.

To extend these results to tidal flows we first disregard the time-variation of tidal flows. We therefore consider a slowly varying tidal flow, such as the spring-neap tide. A tidal flow is forced by water level variations at sea, rather than a fixed discharge velocity. As a consequence, sediment stratification does not reduce the bed shear stress, but instead increases the velocity. We should therefore not consider the existence of multiple equilibria as a function of the velocity, but as a function of the bed shear stress. Fig 1 thus applies to slowly varying tidal flows.

The bed shear stress varies throughout the tidal cycle, so that the sediment concentration moves along the branch of equilibrium solutions as is sketched in Fig 3. If the tide is strong enough, the concentration remains low when progressing from slack to peak tide, then jumps to high concentrations around peak tide and remains high while the flow progresses to slack tide again, before the concentration jumps back. We thus find a hysteresis in concentrations over the tidal cycle.

At the moment we are extending this theory to higher tidal frequencies such as the M_2 frequency and investigate the importance of time-variations. These results will be presented during the conference.

References

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