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Influence of water depth at wave run-up and wave overtopping

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Title
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Summary
Important parameters in the design and assessment of dikes under wave loading are the wave run-up height and the mean wave overtopping discharge. To determine these parameters (empirical) prediction formulas are available. These formulas however have a significant large scatter which does not allow for the most economical design in terms of required crest height.

In the formulas the water depth is not included (exception for extreme shallow foreshores) which is remarkable. In this report a literature study is given which shows that there is an influence of the water depth on the wave run-up height. Also two mechanisms are considered theoretically which support the hypothesis that the water depth does influence the wave run-up height (and therefore also the wave overtopping discharge).

Although it is likely that the water depth does influence the wave run-up and wave overtopping discharge there is not enough basis to incorporate this influence into design formulas. To this end, it is recommended to perform physical model experiments. Depending on the outcome of the experiments, the influence of the water depth can be included in the formulas which will contribute to a more economical design of dikes.

References
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Content

1 Introduction

1.1 General

To protect the Netherlands and other flood prone areas against flooding a thorough understanding of primary flood defences under hydraulic loading is required. An important aspect of dikes under wave loading is wave run-up and wave overtopping. These aspects determine to a large extend the height of the dike. The common failure probability space (in Dutch: *faalkansruimte*) for erosion due to wave overtopping is 24%, indicating the significance of wave overtopping of dikes.

Erosion due to wave overtopping involves the loads and the strength of the dike. This report focuses on the loads which are usually expressed in the mean wave overtopping rate q, which is the mean wave overtopping volume during a considered period over a dike part with a length of 1 m. The unit of q is l/s/m (litre per second per meter dike). This is usually one of the most important parameters that determine the required height of a dike.

To determine the mean wave overtopping rate several empirical formulae are available. A commonly used method is given in TAW (2002) which is also given EurOtop (2007). In that approach the mean wave overtopping rate q is a function of the geometrical aspects of the dike and the hydraulic parameters. Geometrical aspects in the mentioned approach are the slope angle, the presence of berms and crest walls, roughness of the slope and the crest height. The considered hydraulic parameters are the significant wave heights, the spectral wave period and the angle of incident.

Surprisingly the influence of water depth is not taken into account in the given approach. An exception is made for very shallow foreshores where heavy wave breaking, resulting in an entire changed wave spectrum and a lower significant wave height, occurs. In TAW (2002) a ‘very shallow foreshore’ is defined as a foreshore where the wave height is, due to breaking, less than 50% - 60% of its value at deep water.

In 2014 and 2015 several wave run-up tests on different types of block revetments were performed in the Deltares Delta Flume. These tests, where no shallow foreshores were applied, showed that for larger water depths a higher wave run-up was measured. This was also reported in Szmytkiewicz et al (1994) and WL (1993-2). This is remarkable since the water depth is not included in the TAW (2002) formulation which usually is considered as the basis for the design and assessments of dikes with respect to wave run-up and wave overtopping.

The goal of this report is to identify whether the water depth possibly influences the wave run-up and wave overtopping processes and to identify which activities are required to demonstrate and quantify this.

The impact of this research can be relevant for the design and assessments of dikes. This is not only relevant for dikes with a foreshore (or a relatively low water depth) but also for dikes with relatively deep water. It is expected that the incorporation of the water depth in the formulation will lead to better predictions of the wave overtopping characteristics and therefore lead to a more efficient design and assessment of dikes.

1.2 Reading guide

In Chapter 2 the relevance of improving the current wave overtopping formulation is given. This is done by considering the required crest height for a range of parameters and to quantify the spreading around the current overtopping formulas in terms of the required crest height.

In Chapter 3 a literature review is given. Three researches were found were the influence of a foreshore on the wave run-up height was considered.

In Chapter 4 the influence of foreshores on wave run-up due to the breaking of the larger waves is considered. This mechanism leads to a transformation of the Rayleigh distributed waves to none-Rayleigh distributed waves (and likely also to an influence on the wave run-up height distribution).

In Chapter 5 the influence of foreshores is considered based on the so called wave momentum flux theory.

In Chapter 6, conclusions and recommendations are given.

1.3 About this report

This report is written within the framework of the so-called project Kennis voor Primaire Processen (KPP) ‘Versterking Onderzoek Waterveiligheid’.

2 Relevance of improving wave overtopping formulation

2.1 Introduction

In this chapter an insight is given in the uncertainty of the existing wave overtopping formulation. The uncertainty is expressed in terms of required crest height. This is done since that parameter is in several cases one of the most important parameters with respect to the design of dikes. By doing this the relevance of improving wave overtopping formulation will be shown.

2.2 Formulation of wave overtopping according to TAW (2002)

The mean wave overtopping discharge is usually given as:

$$\frac{q}{\sqrt{g \cdot H_{m0}^3}} = \frac{0.067}{\sqrt{\tan\alpha}} \cdot \gamma_b \cdot \xi_0 \cdot e^{-a\frac{R_c}{H_{m0}} \cdot \frac{1}{\xi_0 \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_\upsilon}} \tag{2.1}$$

With a maximum of

$$\frac{q}{\sqrt{g \cdot H_{m0}^3}} = 0.2 \cdot e^{-b\frac{R_c}{H_{m0}} \cdot \frac{1}{\gamma_f \cdot \gamma_\beta}} \tag{2.2}$$

With q is the mean overtopping rate (m^3/s/m), g is the acceleration due to gravity (m/s^2), H_{m0} is the significant wave height (m), R_c is the crest height (m), ξ_0 is the wave breaker parameter based on the spectral wave period $T_{m-1,0}$ (-) and γ_x is a correction coefficient for berms (γ_b), roughness (γ_f), angle of incident (γ_β) and crest walls (γ_v).

The reliability of Eq. (2.1) and Eq. (2.2) is given by describing the coefficient a (μ = 4.75, σ = 0.5) and b (μ = 2.6, σ = 0.35) as normally distributed stochastic parameters. The spreading around Eq. (2.1) is visualised in Figure 2.1. In TAW (2002) a ‘safe’ recommended value of a = 4.25 is given which is 1 standard deviation from the mean (in 15.9% of the cases more overtopping will occur).

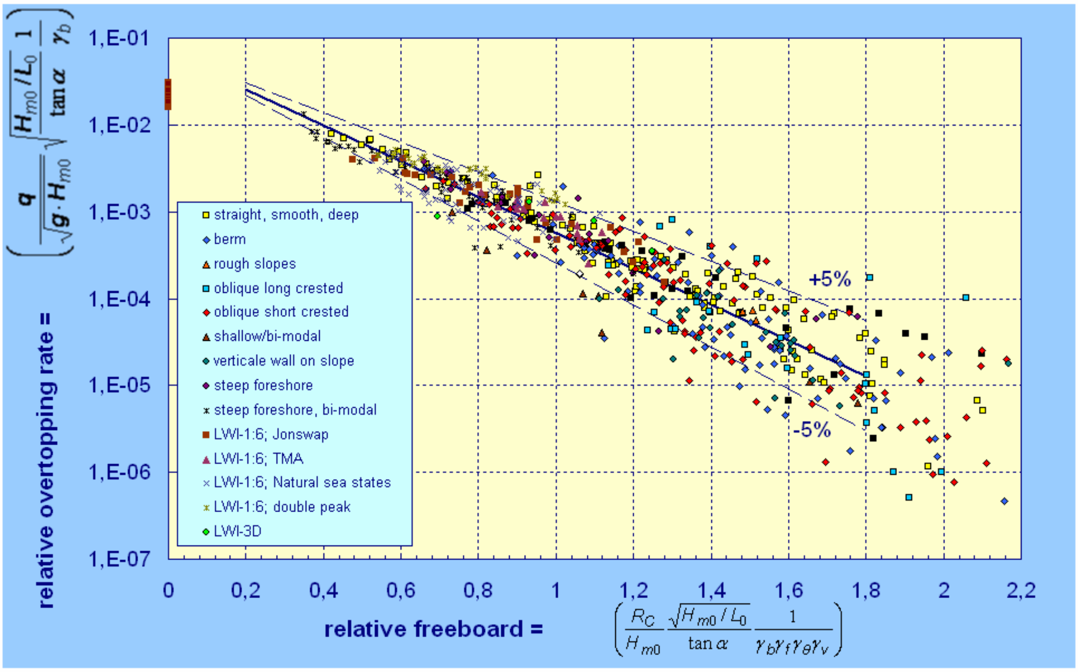


Figure 2.1 Wave overtopping data for breaking waves with 5% under and upper exceedance limits based on Eq.(2.1) (EurOtop, 2007)

The horizontal axis of Figure 2.1 can be read as follows: at the left side of the axis

- the crest height (R_c) is relatively low and / or;
- the significant wave height (H_{m0}) is relatively high and / or;
- the wave length (L_0) is relatively large and / or;
- the slope angle (α) is relatively steep and / or;
- the values of the influence factors (γ_x) are relatively high.

The vertical axis of Figure 2.1 can be read as follows: at the lower side of the axis:

- the mean wave overtopping rate (q) is relatively low and / or;
- the significant wave height (H_{m0}) is relatively high and / or;
- the wave length (L_0) is relatively large and / or;
- the slope angle (α) is relatively steep and / or;
- the value of influence factor of berms (γ_b) is relatively high.

The uncertainty can be expressed in terms of crest height (R_c) given fixed values for all other parameters. This parameter is chosen since the crest height is usually the main interest. To do this we have to choose an upper and lower uncertainty value. Examples of the value of *a* and *b* as function of the exceedance level are given in Table 2.1.

Table 2.1 Values of parameter a and b as function of the exceedance level

Parameter	μ	σ	0.1%	1%	5%	15.9%*	50%	84.1%	95%	99%	99.9%
						1S		1S			
a	4.75	0.50	3.21	3.59	3.93	4.25	4.75	5.25	5.58	5.92	6.30
b	2.60	0.35	1.52	1.78	2.02	2.25	2.60	2.95	3.18	3.42	3.68

* 'recommended' formula according to TAW (2002)

2.3 Uncertainty expressed in crest height

In the previous section the wave overtopping formulation including uncertainty is given. In this section the uncertainty is expressed in the crest height. For simplicity reasons only Eq. (2.1) is considered. Rewriting Eq. (2.1), and isolating the crest height R_c, gives:

$$R_c = \frac{H_{m0} \cdot \xi_0 \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \gamma_\upsilon}{-a} \ln\left(\frac{q}{\sqrt{g \cdot H_{m0}^3}} \cdot \frac{\sqrt{\tan\alpha}}{0.067} \cdot \frac{1}{\gamma_b \cdot \xi_0} \right) \quad (2.3)$$

Now the required crest level is given as function of the geometric parameters, the load parameters and the uncertainty parameter. For illustration purposes the following situation is assumed:

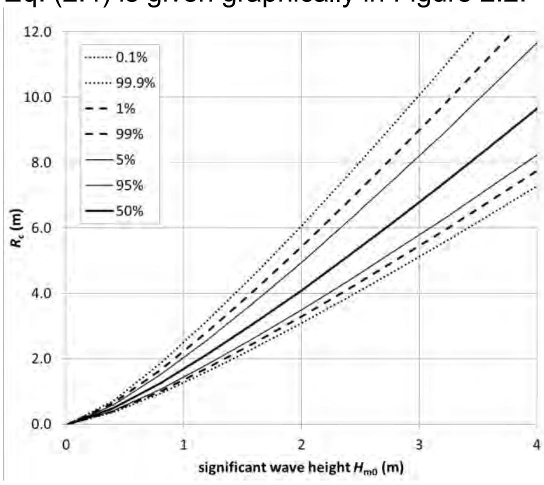
Significant wave height:	0 m	<	H_{m0}	<	4 m
Slope angle			$\cot\alpha$	=	3
Wave steepness			s_0	=	0.04
Influence factor			γ_f	=	$\gamma_b = \gamma_\beta = \gamma_v = 1$
Allowable overtopping rate			q	=	5 l/s/m
Uncertainty level					{0.1; 1; 5; 50; 95; 99; 99.9} %

The given values are chosen since they are representative for many Dutch dikes.

Eq. (2.3) can now be rewritten as:

$$R_c = f(H_{m0}, a) \quad (2.4)$$

Eq. (2.4) is given graphically in Figure 2.2.



Exc. /Hm0	1.0	2.0	3.0	4.0
0.10%	2.5	6.1	10.1	14.3
1%	2.2	5.4	9.0	12.8
5%	2.0	5.0	8.2	11.7
15.9% (TAW)	1.9	4.6	7.6	10.8
50%	1.7	4.1	6.8	9.7
95%	1.4	3.5	5.8	8.2
99%	1.4	3.3	5.4	7.8
99.9%	1.3	3.1	5.1	7.3

Figure 2.2 Required crest height R_c (in m) as function of significant wave height H_{m0} (m) and exceedance level (for s_0 = 0.04, $\gamma_f=\gamma_b=\gamma_\beta=\gamma_v=1$, q = 5 l/s/m.

In Figure 2.2 the following aspects can be seen (for the given parameters):

- The required crest height (R_c) is given as function of the significant wave height (H_{m0})
- A larger value of the significant wave height H_{m0} leads to a larger required crest height R_C.
- The spreading around the estimated required crest height ('50%') is given as well ('0.1%', '1%', '5%', '95%'. '99.9%') and visualises the uncertainty around the estimated value (expressed in a crest height).

- The uncertainty (expressed as a crest height R_c) is larger for larger values of the significant wave height H_{m0}
- Example: for H_{m0} = 3 m, the uncertainty is as follows:
  - The 90% interval (5%-95%, solid lines) is equal to: 5.8 m ≤ R_c ≤ 8.2 m.

2.4 Potential crest height reduction

Suppose it is possible to optimize the wave overtopping formulation in a ‘perfect’ way resulting in a model where no uncertainty exists (σ_a = 0.00). Of course this is a utopia but it does illustrate the potential improvement in wave overtopping modelling. This perfect model can now be compared with the TAW (2002) recommended formula (*a* = 4.25). It is assumed that the results of the perfect model are distributed according to the probabilistic approach given in TAW (2002) but are known for each specific case. What is now the profit (or losses) in terms of the height of the crest level?

On average the profit is (indirectly) the safety factor given in the TAW (2002). This safety factor, one standard deviation, is included in the factor *a* (*a* = 4.25 for recommended approach, for probabilistic approach: μ_a = 4.75, σ_a = 0.5). Suppose a situation with a significant wave height of H_{m0} = 3.0 m (and all other parameters with values as given in Section 2.3). With *a* = 4.25 (15.9%), the required crest height is 7.6 m, with *a* = 4.75 (50%) the required crest height is 6.8 m. The *expected* profit (less crest height) is therefore equal to 7.6 m - 6.8 m = 0.8 m (red dot in Figure 2.3).

However, in 50 % of the cases the expected profit will be less and in 50% of the cases the expected profit will be more. For example, in 5% of the cases the required crest height is 5.8 m; the profit is (7.6 m - 5.8 m) = 1.8 m (green dot in Figure 2.3).

In 16% of the cases the required crest level is higher than predicted with the recommended TAW (2002) formula; this could be considered as an unsafe situation, which is illustrated with the red rectangular in Figure 2.3. In 84% of the cases the crest height is over dimensioned, which is illustrated with the blue rectangular in Figure 2.3.

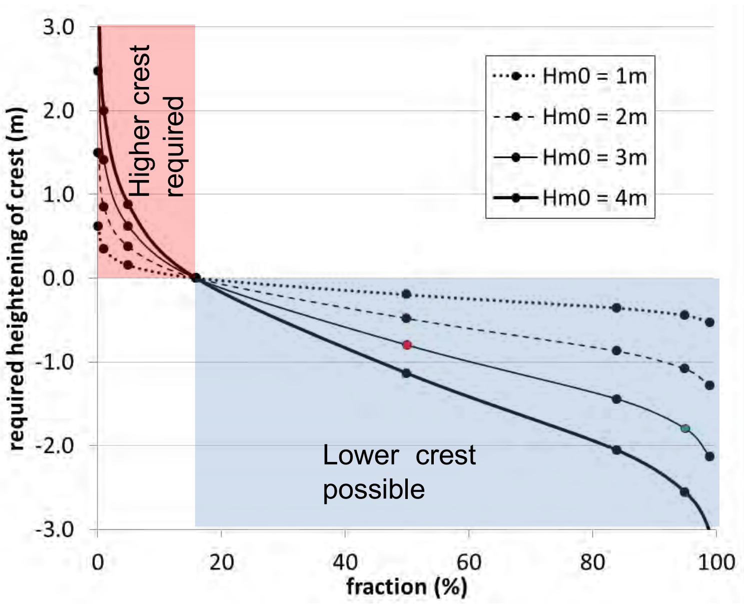


Figure 2.3 Required heightening of dike crest when using probabilistic model (μ_a = 4.75, σ_a = 0.5) compared to deterministic model (a = 4.25). Figure is only valid for cotα = 3, s_{op} = 0.04, q = 5 l/s/m, $\gamma_f=\gamma_b=\gamma_\beta=\gamma_v=1$

2.5 Conclusions

It is shown that the uncertainty around the wave overtopping prediction is relatively large when expressed in the crest height R_c. In the given examples it is shown that the potential reduction of the crest height can be significant (several decimetres or in very specific cases even more than 2 m). It is also possible that for some cases the currently used models under predicts the required crest height (several decimetres or in very specific cases more than 1 m).

Reducing the uncertainty of the current wave overtopping formulation will therefore significantly contribute to the optimization of the design of dikes and revetments under wave loads.

3 Previous research on the influence of depth on wave run-up and wave overtopping

3.1 TAW (2002) and underlying research report

Within the Netherlands extensive research has been carried out on wave run-up and wave overtopping. Results are summarized and made applicable in engineering formulas in TAW (2002). In TAW (2002) an overview of the underlying research reports is given which is followed in this chapter. In the literature overview in this chapter only the influence of the depth on wave run-up and wave overtopping is considered.

In (WL-1993-1) a summary of available research with respect to wave overtopping and wave run-up is given. That report was updated in 1997 (WL-1997-1). More background information is given in (WL-1993-2) and (WL-1997-2).

In (WL-1993-1), an influence factor for a shallow foreshore was introduced. The basic idea was that the higher waves will break on the shallow foreshore which results in a different wave height distribution. This is shown in Figure 3.1.

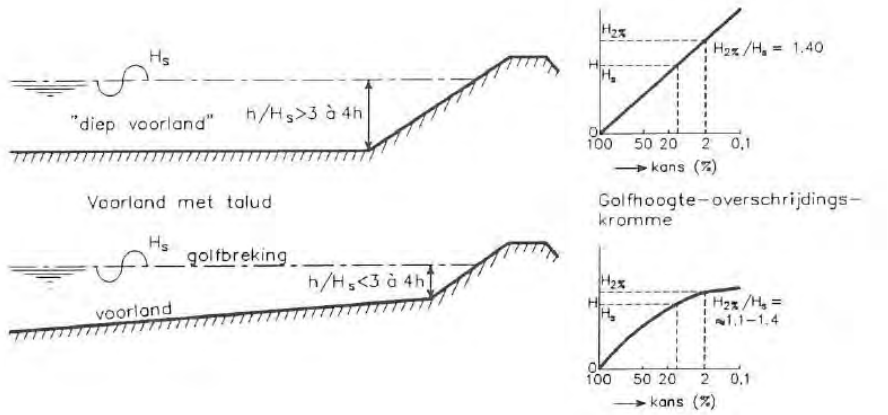


Figure 3.1 Influence of shallow foreshore on wave height distribution (WL-1993-1)

Since the wave run-up height was given as the value that was exceeded by 2% of the waves ($R_{2\%}$) it was assumed that a relation with $H_{2\%}$ was more logical than a relation with the significant wave height (H_s or H_{m0}). For that reason an influence factor γ_h was introduced. This influence factor could be determined by using a given formula. That formula is only valid for a 1:100 slope. The wave height at the position of the toe should be used in the formula. The formula is visualised in Figure 3.2.

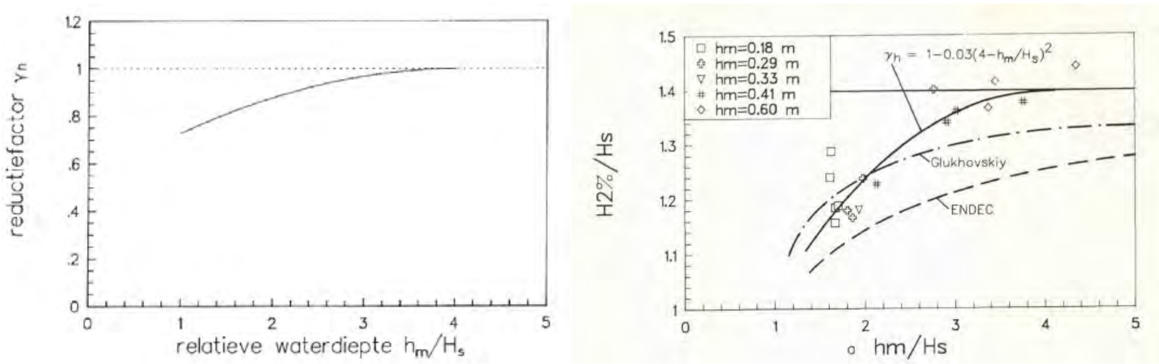


Figure 3.2 left: Reduction factor γ_h for a shallow foreshore (1:100 slope). Right: $H_{2\%}/H_s$ as function of h_m/H_s according to several theories (WL-1993-1)

The following remarks are made:

- The used formula to determine the ratio between $H_{2\%}$ and H_s is only valid for a 1:100 slope. In 2000 a formula to determine the ratio of $H_{2\%}$ and H_s was published (Battjes and Groenendijk, 2000). This is worked out in more detail in Chapter 4.

In (WL 1997-2) it is stated that, based on results of a physical model, an extra influence factor due to shallow foreshores is not justified since there is a large uncertainty in the determination of the wave parameters such as H_{m0} and $H_{2\%}$ in 1997. That conclusion was only valid in situations where the wave height at the toe of the structure is not lower than 60-70% of the wave height at deep water (WL 1997-2 and TAW, 2002). This conclusion is mainly based on a figure which is given in WL-1993-2 and is reprinted as Figure 3.3.

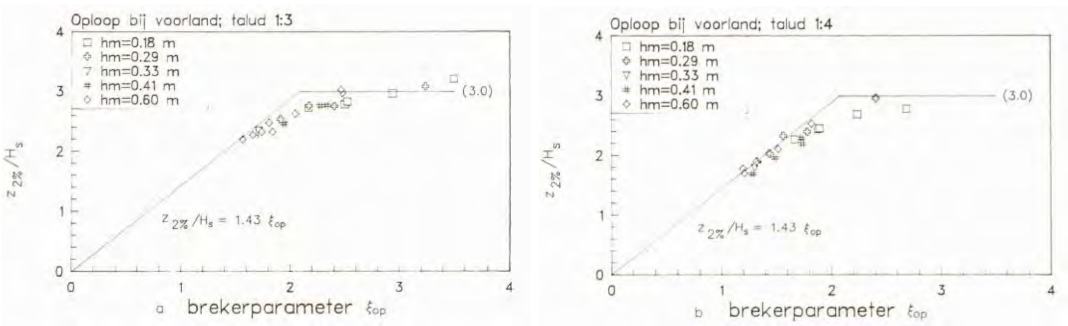


Figure 3.3 Results of tests with shallow foreshores compared with run-up formula that was used in 1993 (without influence of foreshores) (source: WL-1993-2)

VOORLAND		DIEP WATER			ONDIEP WATER		OPLOOP			BEREKENINGEN ONDIEP WATER						OPLOOP	
proef	h_m (m)	h (m)	Hos (m)	Tp (s)	Hsi (m)	H2%i (m)	z2% (m)	z10% (m)	z0.5% (m)	Lo (m)	Hs/Lo (-)	ksi (-)	ksi H2%	HL (m²)	h_m/H (-)	z2%/Hs (-)	z2%/H2% (-)
3301	.18	.53	.181	2.36	.112	.139	.332	.287	.383	8.69	.013	2.94	2.64	.97	1.61	1.24	2.39
3302	.18	.53	.186	1.71	.108	.125	.294	.250	.318	4.56	.024	2.17	2.01	.49	1.67	1.16	2.35
3303	.18	.53	.121	2.80	.111	.143	.356	.300	.395	12.23	.009	3.50	3.08	1.36	1.62	1.29	2.49
3304	.18	.53	.134	1.97	.108	.128	.300	.258	.326	6.05	.018	2.50	2.29	.65	1.67	1.19	2.35
3305	.29	.64	.193	2.09	.161	.190	.444	.373	.489	6.81	.024	2.17	2.00	1.10	1.80	1.18	2.34
3306	.29	.64	.196	1.81	.156	.182	.395	.329	.430	5.11	.031	1.91	1.77	.80	1.86	1.17	2.17
3307	.29	.64	.148	2.21	.147	.182	.404	.339	.434	7.62	.019	2.40	2.16	1.12	1.97	1.24	2.22
3308	.29	.64	.095	2.52	.105	.147	.324	.249	.375	9.91	.011	3.24	2.74	1.04	2.76	1.40	2.20
3309	.33	.68	.220	1.70	.171	.202	.401	.328	.442	4.51	.038	1.71	1.57	.77	1.93	1.18	1.99
3310	.41	.76	.215	1.79	.193	.237	.443	.349	.504	5.00	.039	1.70	1.53	.96	2.12	1.23	1.87
3311	.41	.76	.138	2.10	.141	.189	.391	.310	.446	6.88	.020	2.33	2.01	.97	2.91	1.34	2.07
3312	.41	.76	.146	1.72	.136	.185	.335	.264	.383	4.62	.029	1.94	1.66	.63	3.01	1.36	1.81
3313	.41	.76	.112	1.80	.109	.150	.301	.233	.330	5.05	.022	2.27	1.93	.55	3.76	1.38	2.01
3314	.60	.95	.190	1.86	.178	.243	.414	.318	.485	5.40	.033	1.84	1.57	.96	3.37	1.37	1.70
3315	.60	.95	.194	1.66	.174	.246	.396	.306	.436	4.30	.040	1.66	1.39	.75	3.45	1.41	1.61
3316	.60	.95	.154	1.61	.138	.199	.342	.255	.409	4.04	.034	1.80	1.50	.56	4.35	1.44	1.72
3317	.18	.53	.121	1.97	.106	.126	.300	.248	.323	6.05	.018	2.52	2.31	.64	1.70	1.19	2.38
3318	.60	.95	.178	1.53	.166		.365	.266	.398	3.65	.045	1.56		.61	3.61		
3319	.60	.95	.180	1.70	.166		.386	.296	.478	4.51	.037	1.74		.75	3.61		
3320	.60	.95	.180	2.00	.166		.436	.331	.477	6.24	.027	2.04		1.04	3.61		
3321	.60	.95	.179	2.42	.166		.493	.394	.564	9.14	.018	2.47		1.52	3.61		
3322	.60	.95	.176	2.41	.166		.502	.396	.564	9.06	.018	2.46		1.50	3.61		

Figure 3.4 (Part of) Test results of tests with 1:100 foreshore (only 1:3 slope) (source: WL-1993-2)

As can be seen in Figure 3.4 the ratio between the depth at the toe of the structure (h_m) and the wave height at the toe (H_{si}) varied between 1.61 and 4.35.

Based on current knowledge and insights the following remarks are made:

- The local water depth h_m was chosen at the toe of the structure. In CUR and TAW (1992, Appendix I) it was reasoned that a better representative local water depth is at a certain distance from the toe of the structure since the breaking process will start at a certain point but is not directly entirely broken. In CUR and TAW (1992) a distance of $L_{op}/2$ is suggested but there is no strong basis for this conclusion, especially for shallow foreshore.
- In the analysis the breaker parameter ξ_{op}, which is based on the peak wave period T_p, was used. In TAW (2002) use is made of the spectral parameter $\xi_{m-1.0}$ instead of ξ_{op}. This was mainly based on Van Gent (2001) who showed that the spectral wave period $T_{m-1.0}$ was a more representative parameter than the peak wave period T_p. To compensate for that, all test results were adapted according to a procedure which is given in DWW (2001). However, by studying DWW (2001) it is concluded that no transformation for the shallow water conditions was made since the relation between T_p and $T_{m-1.0}$ for these conditions is unknown.
- The influence of the wave period was not taken into account in the analysis. As will be shown in Chapter 5 this parameter has an important role in the influence of the water depth on the run-up.
- New theories (CWD theory was published in 2000, which is discussed in Chapter 5) that can predict the $H_{2\%}$ value as function of the spectral wave height are developed. This theory was not available during the research in TAW research period (1990-1999).

For deep water conditions with single topped wave energy spectra the theoretical ratio of $T_p/T_{m-1.0}$ = 1.1 was used (DWW, 2001, Chapter 4). Based on that ratio the used data (and prediction formulae) were transformed. Basically all markers and prediction lines in Figure 3.3 shift to the left and the parameter ξ_p on the horizontal axis is replaced by $\xi_{m-1,0}$. A ratio for shallow foreshores is not given but it is known that the ratio $T_p/T_{m-1.0}$ is different from a fixed value like 1.1 ($T_{m-1,0}$ might even become larger than T_p). In that case the markers in Figure 3.3 will have a smaller shift to the left (or even a shift to the right) compared to the deep water situation and compared with the prediction formula. This indicates that there is an influence for the presence of foreshores on wave run-up ($R_{2\%}$).

The knowledge given in several TAW reports is improved with research carried out within WTI2017. For the given subject no specific results were found within the WTI2017 activities.

3.2 Szmytiewicz et al (1994)

In Szmytiewic et al (1994) a dataset with smooth slopes is described. This dataset was created by conducting small scale experiments with H_s = 0.1 m. Szmytiewic et al concluded that the run-up height increases as the water depth increases but also that the run-up height differences were greater when using larger wave periods. The run-up heights as computed by Szmytiewic et al are given in Table 3.1.

Table 3.1 Results of small scale model tests as given by Szmytiewic et al (1994)

H_{os} (m)	T_{op} (s)	h=0.50 m $R^{(2)}_{u2\%}$ (m)	h=0.60 m $R^{(2)}_{u2\%}$ (m)	h=0.70 m $R^{(2)}_{u2\%}$ (m)
0.10	0.90	0.072	0.081	0.088
0.10	1.10	0.096	0.113	0.119
0.10	1.30	0.118	0.140	0.146
0.10	1.50	0.139	0.188	0.181
0.10	1.70	0.155	0.221	0.211
0.10	1.90	0.177	0.213	0.226
0.10	2.10	0.193	0.234	0.252
0.10	2.30	0.206	0.263	0.266
0.10	2.50	0.209	0.260	0.281

The results as given in Table 3.1 are visualised in Figure 3.5.

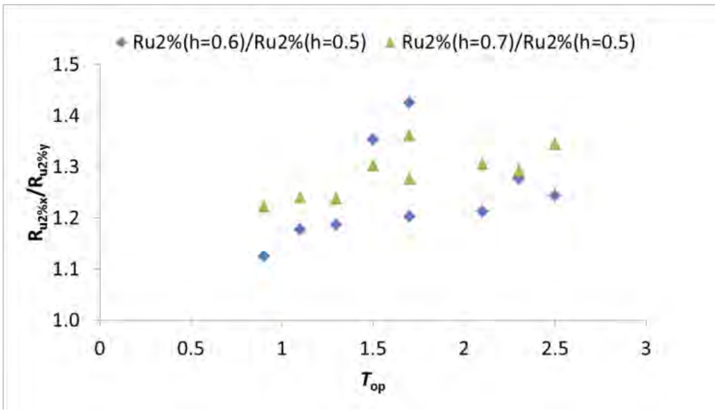


Figure 3.5 Visualisation of results of Szmytiewic et al (1994)

In Figure 3.5 it can clearly be seen that both the water depth and the wave period have a significant influence on the wave run-up height.

3.3 Delta Flume experiments 2015

In 2015 Deltares performed experiments in the Delta Flume to determine the roughness of several types of block revetments. Some experiments were carried out at a different water level. It was observed that the run-up level was lower when applying lower water levels, while having the same revetment and the same wave height and wave period. An impression of the results is given Figure 3.6.

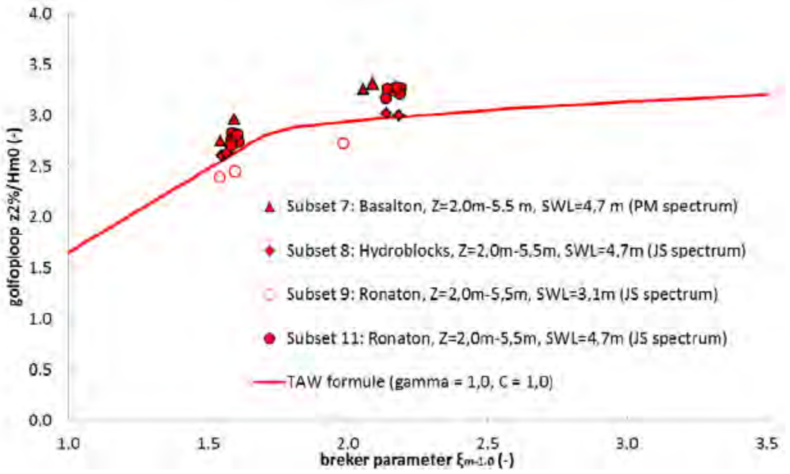


Figure 3.6 Impression of selection of test results obtained with the Deltares Delta Flume in 2015. The open circles represent tests with a relatively low water level, the closed circles are the same type of revetment but with a relatively high water level

3.4 Summary of previous research

In the three described research projects the influence of the water depth is suggested but never worked out in a satisfactory way. At two projects the research was not focussing on the influence of the water depth but had a different research goal. For that reason the influence of the water depth was not worked out in detail. It was however remarked that the water depth did influence the wave run-up height:

- In Szmytiewic et al (1994) (see also Section 3.2) it was explicitly mentioned that a larger water depth led to larger wave run-up heights for equal wave conditions of H_s. T_p;

In Delta Flume experiments in 2015 tests (with equal wave conditions H_s, T_p) were repeated with a lower water level leading to lower run-up heights
In the WL research (1993-1, 1993-2 and 1997-2) it was assumed that only the water depth and the wave height influenced the wave run-up level. Based on limited data it was concluded that there was an influence of the ratio h/H_s on the wave run-up level. This was however not implemented in the TAW (2002) formulation. In the WL research no attention was paid to the influence of the wave period while in Szmytiewic et al (1994) it was suggested that that parameter was important when determining the influence of the water depth.

For that reason both approaches will be worked out in a theoretical way in the following chapters:

- In Chapter 4 only the ratio between the water depth and the wave height is considered (h/H_{m0}). This is based on the wave breaking of the relatively higher waves.

- In Chapter 5 a different approach is used. Based on that approach both the water depth (h) and the wave period (T) are included. This is based on the so-called wave momentum flux parameter.

4 Influence of depth on wave run-up (due to breaking of waves)

4.1 Description of breaking waves as function of the water depth

In this chapter a theoretical description is given of the influence of the water depth on the wave run-up height.

In deep water the wave heights are usually Rayleigh distributed. It is noted that the wave run-up height at smooth slopes is usually also Rayleigh distributed. In shallow water (higher) waves will start to break resulting in a different distribution of the wave height. This is illustrated in Figure 4.1.

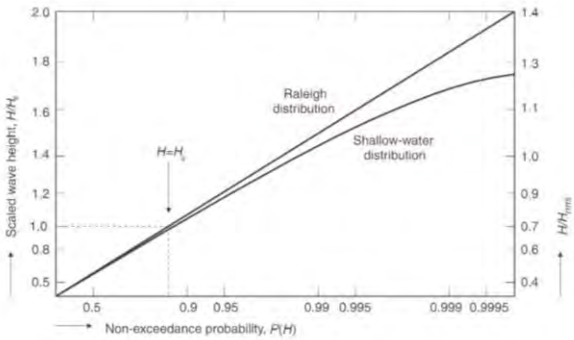


Figure 4.1 Example of a wave height distribution at deep water (Rayleigh) and at shallow water (source: CIRIA et al, 2007)

When breaking occurs, the shape of the wave height distribution is not a Rayleigh distribution anymore. It is likely that also the wave run-up distribution is not Rayleigh distributed anymore resulting in lower wave run-up heights (e.g. 2% value) and lower overtopping quantities.

4.2 Wave run-up as function of the water depth

The wave height distribution of shallow water waves are described in several models such as Hughes and Borman (1987), Glukhovskiy, (1966), Mendez et al (2004) and the so-called Composite Weibull distribution which is described in Battjes and Groenendijk (2000) and Groenendijk and Van Gent (1999). In this report the Composite Weibull distribution is used which makes it possible to determine various exceedance values such as the wave height exceeded by 2% of the waves ($H_{2\%}$). For deep water the ratio between $H_{2\%}$ and H_{m0} is equal to:

$$\frac{H_{2\%}}{H_{m0}} = \left(\frac{H_{2\%}}{H_{m0}}\right)_{\max} = 1.4 \qquad \text{for deep water} \qquad (4.1)$$

For shallow water the ratio between $H_{2\%}$ and H_{m0} is defined as:

$$\frac{H_{2\%}}{H_{m0}} = f \cdot \left(\frac{H_{2\%}}{H_{m0}}\right)_{\max} \qquad \text{for shallow water} \qquad (4.2)$$

Where *f* is a correction factor. Several cases were calculated using the Composite Weibull Distribution. (use was made of the Deltares software tool BREAKWAT). Based on these calculations the following relation was determined for a 1:250 slope:

$$f = 0.0279\left(\frac{h}{H_{m0}}\right)^2 - 0.0781\frac{h}{H_{m0}} + 0.9267 \qquad \text{for } 2 < h/H_{m0} < 3.54 \qquad (4.3)$$

$$f = 1 \qquad \text{for } h/H_{m0} \geq 3.54 \qquad (4.4)$$

With

$H_{2\%}$	= wave height exceeded by 2% of the waves	(m)
H_{m0}	= significant spectral wave height	(m)
h	= water depth	(m)
f	= fraction compared with deep water: $f = (H_{2\%}/H_{m0}) / (H_{2\%}/H_{m0})_{max}$	(-)
$H_{2\%}/H_{m0}$	= relative value of $H_{2\%}$	(-)
$(H_{2\%}/H_{m0})_{max}$	= value of $H_{2\%}/H_{m0}$ for a Rayleigh distributed wave field (value = 1.4)	(-)

The results of the calculation (green markers) and the trendline according to Eq. (4.4) (black line) are visualised in Figure 4.2.

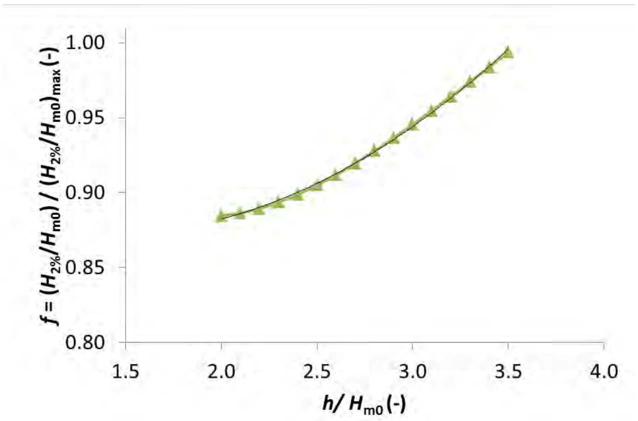


Figure 4.2 Influence of depth on $H_{2\%}$ as calculated with CWD method

It can be seen that in relatively shallow water (h/H_{m0} = 2) the value of $H_{2\%}$ is 88% of the value of $H_{2\%}$ value at deep water (Rayleigh) conditions. It is assumed that the shape of the wave run–up distribution is equivalent to the shape of the wave height distribution. In that case the 2% wave run-up height can be significantly lower for smaller water depths when applying the same significant wave height.

Assuming that the same correction factor *f* can be applied on the 2% wave run–up height $R_{2\%}$, the formulas in TAW (2002) can be corrected. The design formula in TAW (2002) yields:

$$\frac{R_{2\%}}{H_{m0}} = 1.75 \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \xi_0 \qquad (4.5)$$

With a maximum of

$$\frac{R_{2\%}}{H_{m0}} = \gamma_f \cdot \gamma_\beta \cdot (4.3 - 1.6 \cdot \sqrt{\xi_0}) \qquad (4.6)$$

It is assumed that this formula is valid for deep water conditions ($h/H_{m0} \geq 4$), this is however uncertain. When correcting for the influence of the water depth as described above the following formula is obtained:

$$\frac{R_{2\%}}{H_{m0}} = 1.75 \cdot f \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \xi_0 \qquad (4.7)$$

With a maximum of

$$\frac{R_{2\%}}{H_{m0}} = f \cdot \gamma_f \cdot \gamma_\beta \cdot (4.3 - 1.6 \cdot \sqrt{\xi_0}) \qquad (4.8)$$

The result as given in Figure 4.3 illustrates the influence of the relative water depth h/H_{m0}.

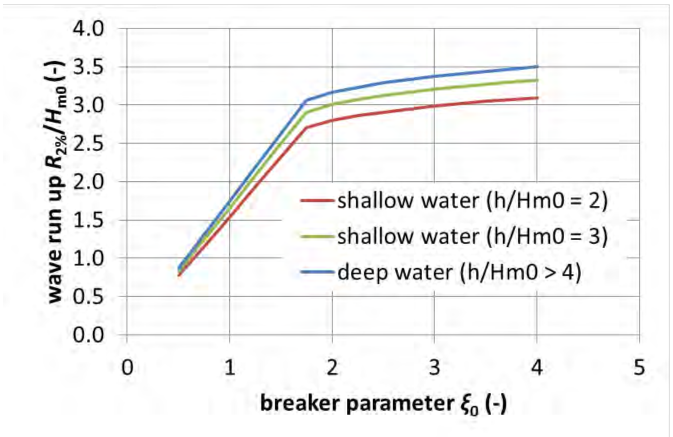


Figure 4.3 Wave run-up as function of breaker parameter for shallow and deep water (based on unverified theory)

4.3 Example

Two situations (Case 1 and Case 2) are considered. The difference between the two situations is the water depth *h*. All parameters are given in Table 5.1.

Table 4.1 Overview of input parameters and calculated parameter of two example cases

parameter			Case 1	Case 2
significant wave height	H_{m0}	(m)	2.5	
water depth	h	(m)	5	10
relative depth	h/H_{m0}	(-)	2	4
breaker parameter	ξ_0	(-)	3	
wave run-up height	$R_{2\%}$	(m)	7.62	8.44

In Case 2 (depth *h* = 10 m) the 2% wave run-up height would be (8.44 m – 7.62 m = 0.82 m higher than the run-up height in Case 1 (depth *h* = 5 m). However, in the current design approach the designer uses the same formula and the same values for both cases resulting in the same run-up value for both cases.

4.4 Conclusions

Based on the above given theory it is likely that the water depth influences the wave run-up heights due to deformation of the wave height exceedance curve. It is stressed that the above given approach is based on severe simplifications. The two most important assumptions are:

- If the ratio of $H_{2\%}/H_{m0}$ of the wave field changes, the 2% wave run-up height $R_{2\%}$ changes as well.
- The influence of the depth on the wave run-up height is independent of the wave period. As discussed in Section 3.2 this is probably not the case.

Despite the two simplifications given above, the analysis given in this Chapter gives sufficient reason to hypothesize that the water depth influences the wave run-up height significantly due to deformation of the wave height exceedance curve.

5 Influence of depth based on wave momentum flux parameter

5.1 Introduction

A schematic representation of the orbital motion of a particle in a wave on a horizontal bed is visualised in Figure 5.1.

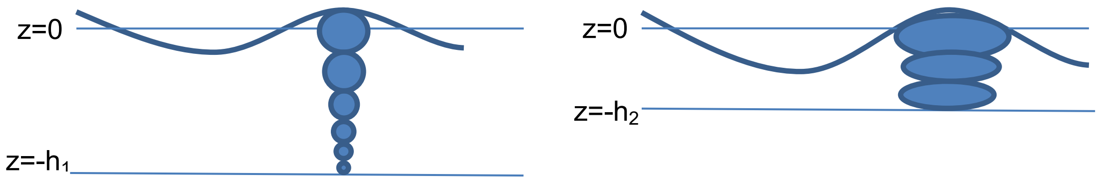


Figure 5.1 Orbital motion of a particle under a wave at 'deep' water (left) and at 'shallow' water (right) (based on linear theory)

The difference of the orbital motion at relatively deep water and relatively shallow water can be seen clearly. The particle velocity can be expressed in a horizontal (u) and a vertical component (w). The maximum velocities (amplitudes) are given by û and ŵ. A schematised representation of the maximum horizontal velocity û for shallow and deep water is given in Figure 5.2.

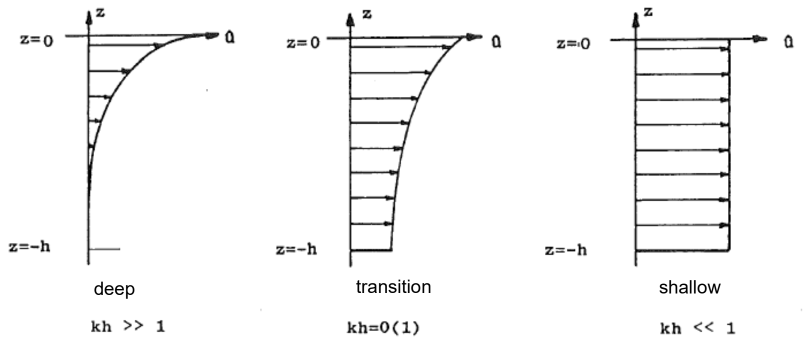


Figure 5.2 Maximum horizontal velocity as function of the vertical position (Battjes, 1990)

It can be seen that the velocity characteristics are considerably different. Also the shape of the wave can change considerable, which is illustrated in Figure 5.3.

Figure 5.3 Different shape of deep water wave (left) and shallow water wave (right) (Battjes, 1990)

A different wave shape gives different relations for the water motion in the waves. An illustration of the different wave theories as function of the relative water depth and the relative wave steepness is given in Figure 5.4.

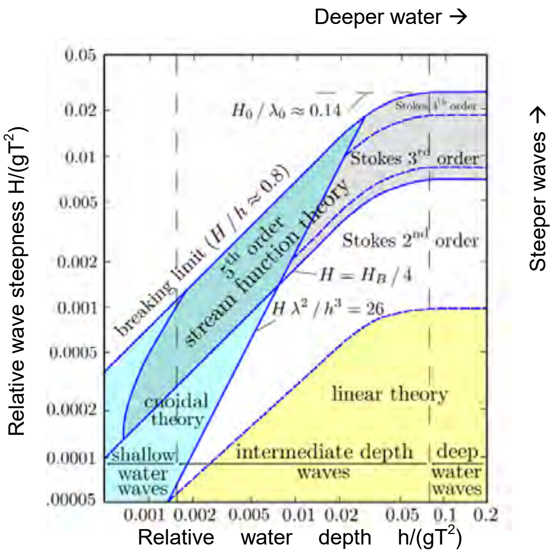


Figure 5.4 Wave theories (after Le Mehauté, 1976)

It is shown that the shape of the wave and the particle motion change with different relative wave steepness and different relative water depth. Therefore it is likely that these aspects influence the wave run-up and wave overtopping characteristics. To study this in more detail the so called 'wave momentum flux parameter' will be introduced in the following section.

5.2 Description of the maximum wave momentum flux parameter

Hughes (2004a,b) suggested to use the maximum wave momentum flux parameter as a descriptor for nearshore waves. The momentum flux parameter can be seen as a property of the wave which is close to force loads on solid structures placed in the wave field and is considered as a physical sound parameter for wave run-up and wave overtopping processes.

The approach is based on non-linear (Fourier) theories.

The basic idea is that the weight of the area ABC (see Figure 5.5) should be proportional to the maximum depth integrated momentum flux or, according to Hughes:

$$K_p (M_f)_{\max} = K_M W_{(ABC)} \tag{5.1}$$

With

K_p = reduction factor to account for slope porosity (K_p = 1 for impermeable slopes)
K_M = unknown constant of proportionality
W = weight of the fluid in the considered area
$(M_f)_{max}$ = maximum wave momentum flux

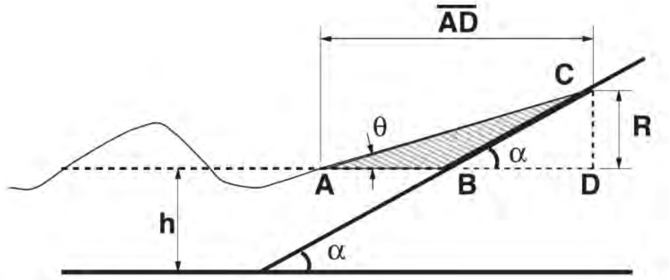


Figure 5.5 Schematisation of maximum wave run-up on a smooth impermeable slope (Hughes (2004b))

More background information about the wave momentum flux parameter is given in Appendix A. Based on the analysis given in Appendix A the run-up height is a function of the momentum flux parameter $M_f/(\rho g h^2)$ and the water depth *h*:

$$R = h \cdot C \cdot F(\alpha) \cdot \left(\frac{M_f}{\rho g h^2} \right)^{1/2} \tag{5.2}$$

With *R* is the wave run-up height, *h* is the water depth, *C* is a constant, *F*(*α*) is a function of the slope angle to be determined empirically and $M_f/(\rho g h^2)$ is the wave momentum flux parameter. The wave momentum flux parameter is dependent on the relative depth (h/gT^2) and relative wave height (*H*/*h*) as given in Figure 5.6 and described by:

$$\frac{M_f}{\rho g h^2} = A_0 \left(\frac{h}{gT^2} \right)^{-A_1} \tag{5.3}$$

$$A_0 = 0.6392 \left(\frac{H}{h} \right)^{2.0256} \tag{5.4}$$

$$A_1 = 0.1804 \left(\frac{H}{h} \right)^{-0.391} \tag{5.5}$$

As can be seen in Eq. (5.2) to Eq. (5.5), the wave run-up height *R* is amongst others a function of the water depth *h*.

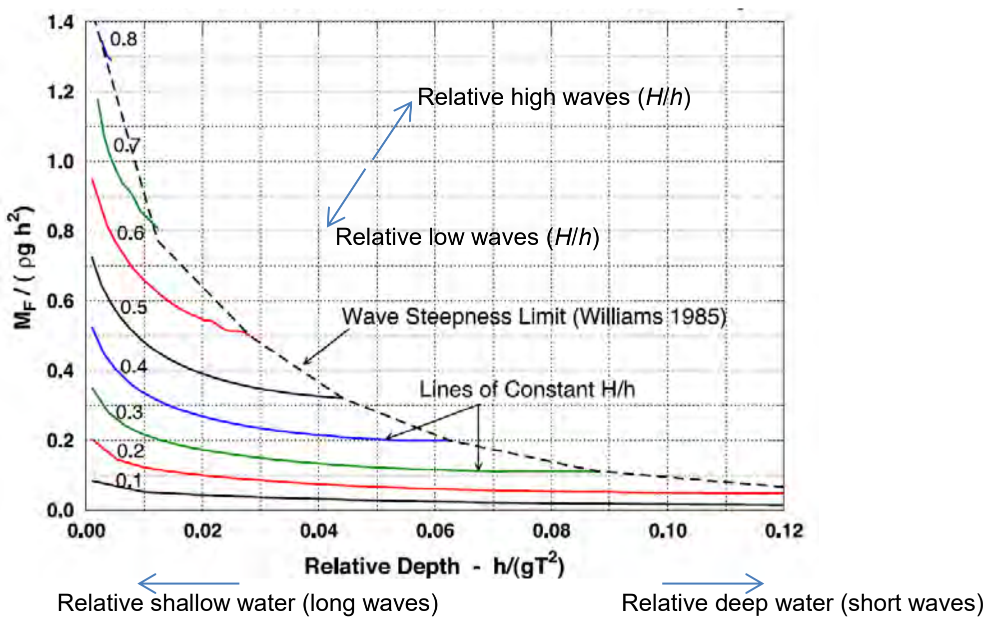


Figure 5.6 Wave momentum flux parameter as function of $h/(gT^2)$ and H/h (after Hughes (2004a))

5.3 Wave run-up as function of the water depth

When using Eq. (5.2) to Eq. (5.5), several plots can be made. First, a plot of the run-up height *R* as function of the wave height *H*, the wave steepness *s*, and the water depth is given in Figure 5.7.

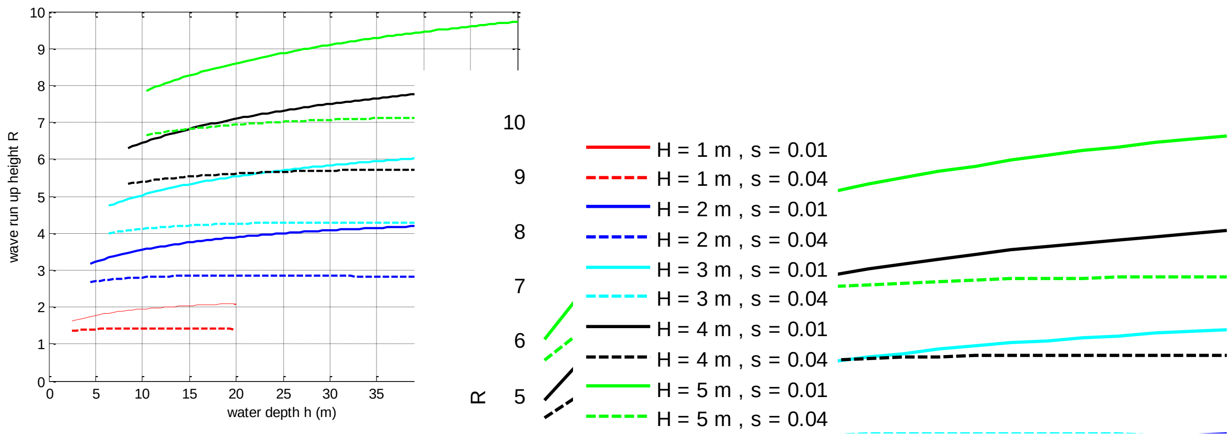


Figure 5.7 Wave run-up height R as function of wave height H, wave steepness s and water depth h

In Figure 5.7, it can clearly be seen that for larger water depths *h*, the run-up height *R* will be larger. This is stronger for relatively long waves (small value of *s*, indicated with a dotted line). In Figure 5.8 the run-up height *R* (vertical axis) and the depth *h* (horizontal axes) are made dimensionless with the wave height *H*.

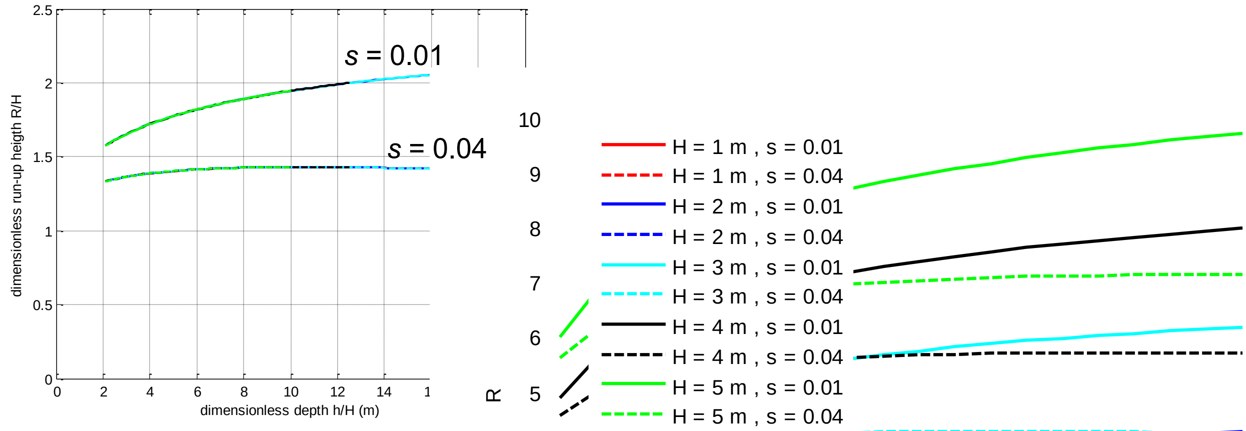


Figure 5.8 Dimensionless wave run-up height R/H as function of wave height H, wave steepness s and water depth h

In Figure 5.8 it can be seen that the dimensionless parameter *h*/*H* and *R*/*H* are suitable dimensionless parameters since the lines with equal wave steepness are equal. It is however emphasized that the wave steepness is also an important parameter.

5.4 Example

Two situations (Case 1 and Case 2) are considered. The difference between the two situations is the water depth *h*. All parameter are given in Table 5.1.

Table 5.1 overview of input parameters and calculated parameter of two example cases

parameter			Case 1	Case 2
wave height	H	(m)	2.5	
wave steepness	s	(-)	0.01	
water depth	h	(m)	7.5	15
constant (assumed)	C	(-)	1	
constant (assumed)	$F(\alpha)$	(-)	1	
wave period	T	(s)	12.65	
deep water wave length	L_0	(m)	250	
local wave length	L	(m)	105	250
relative depth	$h/(gT^2)$	(-)	0.0048	0.0096
relative wave height	h/H	(-)	3	6
momentum flux parameter	$M_f/(\rho g h^2)$	(-)	0.304	0.092
run-up height	R	(m)	4.13	4.55

In Case 2 (depth h = 15 m) the wave run-up height would be (4.55 - 4.13) / 4.13 = 0.1 = 10% higher than the run-up height in Case 1 (depth h = 7.5 m).

5.5 Conclusions

Following the physical sound approach of Hughes (2004a,b), by using the wave momentum flux parameter, it is concluded that the wave run-up is dependent on the water depth. This dependency is smaller for larger wave steepness. Quantification of this influence on the wave run-up height is difficult since many assumptions are made (for example assumed values for constants such as C and $F(\alpha)$) but differences in the order of magnitude of 10% seem likely.

The theory is not projected on irregular wave fields. The influence described in this chapter is a different influence than the mechanism described in Chapter 4 were wave breaking is considered.

6 Conclusions and recommendations

Based on a recent research project in the Delta Flume it is concluded that it is likely that the water depth influences the wave run-up and wave overtopping characteristics significantly. This is also supported by older research (Szmytkiewicz et al, 1994) and theoretical approaches given in this report.

Two mechanisms that potentially influence the wave run-up height are:

- Mechanism 1: Wave breaking of higher waves due to shallow foreshore leading to a non-Rayleigh wave height distribution and therefore a non-Rayleigh wave run-up height distribution. Decreasing depth will therefore result in a lower wave run-up height.
- Mechanism 2: A lower wave momentum flux leads to a lower wave run-up height. This process is a function of the water depth and the wave steepness.

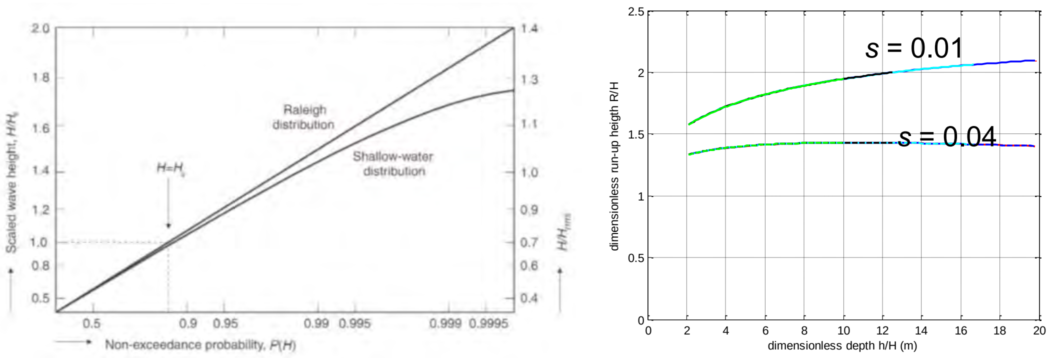


Figure 6.1 Left: Basic principle of Mechanism 1 (Rayleigh distribution is affected due to shallow foreshore). Right: Basic principle of Mechanism 2 (the wave shape is different due to the presence of a shallow foreshore. The resulting wave run-up height is a function of the wave steepness and the depth)

Both mechanisms are supported by theoretical approaches which are given in this report (Chapter 4 and Chapter 5). This is only worked out for wave run-up and not for wave overtopping discharge. In the theoretical approach several assumptions were made.

In literature several cases were found where it was suspected that the water depth influenced the results. However, since this influence was not the main scope of the research this was not worked out in much detail in these studies.

Reducing the spreading around common wave run-up and wave overtopping formulas will lead to a more economical way of designing water defences under wave attack. Incorporation of the two mechanisms as identified in this report might lead to an improved formula and therefore contribute to a more economical design of dikes.

It is however stated that the theories given in this report do not give a sufficient basis to adapt the current existing formulas. Therefore, it is suggested to perform physical model tests that focus on the mechanisms as identified. Although the study in this report is focused on wave run-up it is recommended to focus the physical modelling on wave overtopping discharge since this is the commonly used parameter that is regarded in the design and assessment of dikes.

Performing a physical model study may improve the wave overtopping model leading to a more efficient way of the design and assessment of dikes.

It is recommended to investigate how potential outcomes of a physical model can be implemented in the current empirical relations. It is recommended to consider a prediction based on that model prior to testing.

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A Wave moment flux parameter

A.1 Introduction to the wave flux parameter

The theory below is derived from Hughes (2004a,b).

The instantaneous flux of horizontal momentum across a vertical plane at a certain position (*x*,*z*) at a certain time (*t*) is given by:

$$m_f(x,z,t) = p_d + \rho u^2 \tag{A.1}$$

Where

m_f	=	instantaneous flux of horizontal momentum across a unit area of a vertical plane oriented parallel to the wave crest
p_d	=	instantaneous wave dynamic pressure at a specified position
u	=	instantaneous horizontal water velocity at the same specified position
ρ	=	water density

The maximum depth integrated wave momentum flux parameter during the passage of a wave is derived by using the following expression:

$$M_f(x,t) = \int_{z=-h}^{z=\eta(x)} (p_d + \rho u^2) dz \tag{A.2}$$

Which has to be determined for η(x) = *a,* with *a* = the wave amplitude. To solve this integral Hughes (2004a,b) used three theories:

1. Linear (first order) wave theory: integration from $z = -h$ to $z = 0$ (is still water line)
2. Extended linear wave theory: integration from $z = -h$ to $z = a$ (crest of wave)
3. Nonlinear (Fourier) wave theory: integration from $z = -h$ to $z = a$ (crest of wave) using nonlinear wave theory

The linear wave theory assumes linear shaped waves and neglects the momentum flux above the still water line. The extended linear wave theory also assumes linear shaped waves but takes the momentum flux above the still water line into account. The nonlinear wave theory includes the effects of non-sinusoidal wave forms.

The nonlinear wave theory is the best approximation since it includes both the momentum flux above the still water line and the non-sinusoidal shape of the wave. Results of the nonlinear wave theory are given in
Figure A.1. In that graph the maximum momentum flux is made dimensionless by $\rho g h^2$ and given as function of H/h and $h/(gT^2)$. The limiting wave steepness is given by using the wave-breaking criterion as tabulated by Williams (1985).

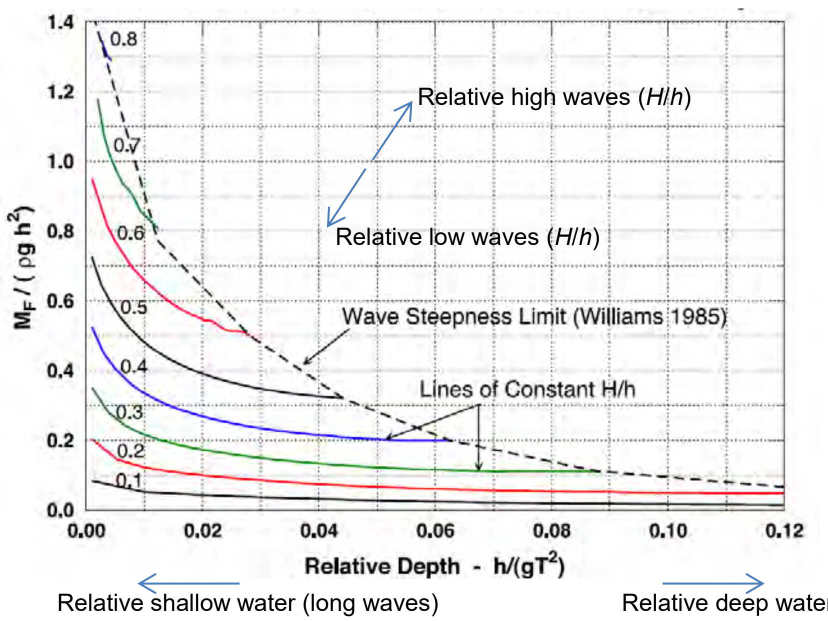

Figure A.1 Wave momentum flux parameter as function of h/(gT²) and H/h (after Hughes (2004a))

A disadvantage of the nonlinear wave theory is that the momentum flux must be calculated numerically. Therefore, Hughes (2004a) proposed empirical equations representing the lines given in Figure A.1:

$$\frac{M_f}{\rho g h^2} = A_0 \left(\frac{h}{gT^2} \right)^{-A_1} \quad \text{(A.3)}$$

$$A_0 = 0.6392 \left(\frac{H}{h} \right)^{2.0256} \quad \text{(A.4)}$$

$$A_1 = 0.1804 \left(\frac{H}{h} \right)^{-0.391} \quad \text{(A.5)}$$

Now it is possible to determine the dimensionless maximum wave momentum flux parameter as function of the dimensionless parameters H/h and $h/(gT^2)$. As can be seen, the water depth h is included in the formulation indicating the relevance of the water depth on the momentum flux and therefore potentially also on the wave run-up and wave overtopping characteristics.

A.2 Wave run-up as function of the wave momentum flux parameter

Hughes (2004b) assumes a wave run-up shape such as given in Figure A.2. The weight of the area ABC should be proportional to the maximum depth integrated momentum flux or, according to Hughes:

$$K_p (M_f)_{\max} = K_M W_{(ABC)} \quad \text{(A.6)}$$

With

K_p = reduction factor to account for slope porosity (K_p = 1 for impermeable slopes)
K_M = unknown constant of proportionality

W = weight of the fluid in the considered area
$(M_f)_{max}$ = maximum wave momentum flux

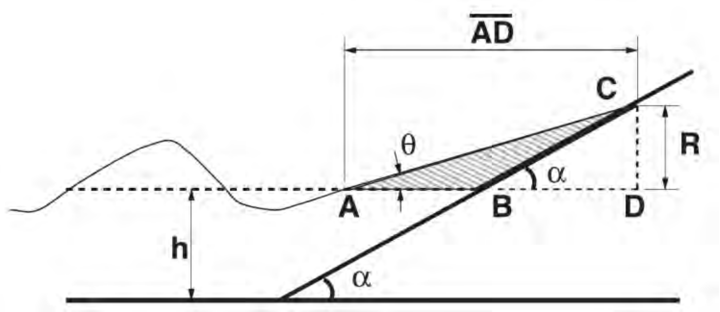

Figure A.2 Schematisation of maximum wave run-up on a smooth impermeable slope (Hughes (2004b))

For a complete derivation reference is made to Hughes (2004b). The resulting equation following rewriting Eq. (A.6) is:

$$\frac{R}{h}=\left(\frac{2K_p\tan\alpha}{K_M\left[\frac{\tan\alpha}{\tan\theta}-1\right]}\right)^{1/2}\cdot\left(\frac{M_f}{\rho g h^2}\right)^{1/2} \quad \text{(A.7)}$$

Which is rewritten by Hughes as:

$$\frac{R}{h}=C\cdot F(\alpha)\cdot\left(\frac{M_f}{\rho g h^2}\right)^{1/2} \quad \text{(A.8)}$$

With:

C = unknown constant

$$C=\left(\frac{2K_p}{K_M}\right)^{1/2} \quad \text{(A.9)}$$

$F(\alpha)$ = function of slope angle to be determined empirically or

$$F(\alpha,\theta)=\left(\frac{\tan\alpha}{\tan\alpha-\tan\theta}\right)^{1/2} \quad \text{(A.10)}$$

Eq. (A.8) can be rewritten as:

$$R=C\cdot F(\alpha)\cdot\left(\frac{M_f}{\rho g}\right)^{1/2} \quad \text{(A.11)}$$

Or, when maintaining the momentum flux parameter $M_f/(\rho g h^2)$:

$$R = h \cdot C \cdot F(\alpha) \cdot \left(\frac{M_f}{\rho g h^2} \right)^{1/2} \quad \text{(A.12)}$$

Now the run-up height is described as a function of the momentum flux parameter $M_f/(\rho g h^2)$ and the water depth h.