waterloopkundig laboratorium
delft hydraulics laboratory

model for sedimentation predictions

L.C. van Rijn

publication no. 241

November 1980

model for sedimentation predictions

prepared for the 19th IAHR-congress,
New Delhi, India, February 1981

L.C. van Rijn

publication no. 241

November 1980

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH
SEDIMENTATION ENGINEERING FOR RIVERS AND RESERVOIRS
(Subject A.d.)

Model for Sedimentation Predictions

by

L.C. van Rijn
Project Engineer, Delft Hydraulics Laboratory, The Netherlands

Synopsis

The paper describes a mathematical model for sedimentation predictions in dredged trenches, sediment traps and navigation channels in rivers or estuaries. The model simulates the suspended sediment transport by diffusion, convection and gravity. Also the convection by the vertical flow velocities is taken into account. The vertical distribution of the sediment, diffusion coefficient is assumed to be parabolic-constant.

In case of sedimentation in a trench perpendicular to the flow the longitudinal flow velocities are computed by an empirical model, which can also predict flow separation and reversed flow velocities (steep-sided trenches). The vertical flow velocities are determined from the equation of continuity. The diffusion-convection equation is solved by an implicit finite-difference method. To model the large concentration gradients near the bed and to obtain a rectangular grid pattern for each bottom configuration a transformation method is used.

To verify the model, laboratory and field data concerning the sedimentation in a trench, are used. The influence of the main controlling parameters on the sedimentation process is also examined.

The model is restricted to non-cohesive, uniform or moderately graded bed material.

Resumé

L'article décrit un modèle mathématique pour des prédictions de sédimentation dans une trenchée draguée, une prise de sable, ou des chenaux de navigation situés dans des fleuves et des estuaires. Le modèle peut simuler le mouvement des sédiments en suspension par la diffusion, la convection et la gravité. La convection par les vitesses de courant verticales est integrée aussi. La distribution verticale du coéfficient de diffusion est supposée d'être parabolique-constante.

En cas de sédimentation dans une trenchée perpendiculaire sur la direction de courant les vitesses longitudinales sont calculées à l'aide d'un modèle empirique prédisant aussi la séparation de courant et des vitesses renversées (des trenchées à des pentes raides). Les vitesses verticales sont determinées par l'équation de continuité. L'équation de diffusion-convection est resolue par une méthode numériques (implicite) de différences limitées.

Pour modéler les gradients larges de concentrations près du lit et pour obtenir un système numérique rectangulaire, l'équation de diffusion-convection est transformée.

Le modèle mathématique est verifié par des essais en laboratoire et des données de mesures en nature concernant la sédimentation dans une trenchée. L'influence des paramètres les plus importants sur le phénomène de sédimentation est examinée aussi.

Le modèle est limité aux matériaux non-cohésifs, et pratiquement uniformes.

1. Introduction

Earlier studies on mathematical modelling of suspended sediment transport for rivers and estuaries carried out by the Delft Hydraulics Laboratory [1], [2] have shown the applicability of mathematical models for sedimentation predictions in dredged trenches (perpendicular to the flow direction) with gentle side slopes. The present paper considers some modifications with respect to the sedimentation in steep-sided trenches, which may be preferable because of reduced dredging works.
A similar model can be used for sedimentation predictions in navigation channels.

2. Basic equations

Water flow

In case of a steep-sided trench the flow and turbulence field is very complex, particularly if flow separation does occur. To describe the longitudinal and vertical flow velocities an empirical model, based on extensive laboratory measurements, was developed. In all, sixteen tests with varying trench dimensions and hydraulic conditions were carried out [3]. A Laser-Doppler velocity meter was used to determine the mean flow velocities, the root-mean-square value of the turbulent velocity fluctuations and the turbulent shear stresses. Static pressures were also measured.
To describe the flow field in a dredged trench, three characteristic zones are distinguished: a deceleration zone where a mixing layer and a reversed flow layer in case of flow separation will be formed, a relaxation zone where the mixing layer will change to a new boundary layer and an acceleration zone where the flow will be accelerated [3].
In mathematical form:

$$u(z) = F(\text{empirical data}) \qquad (1)$$

$$w(z) = w(z = z_b + h) + \int_{z}^{z_b + h} \frac{\delta u}{\delta x}\, dz \text{ (continuity)} \qquad (2)$$

in which: u = longitudinal flow velocity, w = vertical flow velocity, x = longitudinal coordinate, z = vertical coordinate, z_b = bed level above a datum, h = flow depth.

Sediment flow

For (quasi-) steady, flow conditions the vertical and longitudinal movement of the suspended sediment can be described by:

$$\frac{\delta(uc)}{\delta x} + \frac{\delta\{(w - w_s)c\}}{\delta z} - \frac{\delta}{\delta z}\left(\varepsilon_s \frac{\delta c}{\delta z}\right) = 0 \qquad (3)$$

---

1. Kerssens, P.J.M., Rijn, L.C. van, and Wijngaarden, N.J. van,
   Model for Non-Steady Suspended Sediment Transport,
   Paper A15, I.A.H.R. Congress, Baden-Baden, Germany, 1977.

2. Kerssens, P.J.M., Prins, A. and Rijn, L.C. van,
   Model for suspended Sediment Transport,
   Journal of the Hydraulic Division, A.S.C.E., May 1979.

3. Delft Hydraulics Laboratory,
   Semi-Empirical Model for the Flow in Dredged Trenches,
   Report R 1267-III/M 1536, 1980.

in which: c = sediment concentration, w_s = particle fall velocity, ε_s = sediment diffusion coefficient.

The diffusion coefficient is assumed to be a scalar quantity, while the particle fall velocity is assumed to be constant in vertical direction.
By means of scale analysis it can be shown [2] that the longitudinal diffusion term is negligibly small with respect to the other terms.

Bed level changes
Bed level changes can be computed from the equation of continuity for the total sediment transport:

$$\frac{\delta z_b}{\delta t} + \frac{1}{(1-p)\rho_s} \frac{\delta s_t}{\delta x} = 0 \tag{4}$$

in which: s_t = total sediment transport, p = porosity coefficient, ρ_s = sediment density.

The total sediment transport is considered to be the sum of the depth-integrated suspended sediment transport and the bed load transport. The bed load transport may be estimated by a simple formula which relates the bed load transport to the depth-averaged flow velocity and the sediment size. Gravity-effects on the bed load particles at the side slopes of the trench are not represented in the mathematical model.

3. Sediment diffusion coefficient

Uniform flow
Outside the trench and in case of a trench with gentle side slopes the flow is assumed to be nearly-uniform so that the diffusion coefficient can be represented by a (parabolic-constant) distribution for uniform flow [1], [2]:

$$\varepsilon_s = 4 \left(\frac{z-z_b}{h}\right) \left\{1 - \left(\frac{z-z_b}{h}\right)\right\} \varepsilon_{s,\max}, \quad \text{for} \quad \frac{z-z_b}{h} < 0.5 \tag{5}$$

$$\varepsilon_s = \varepsilon_{s,\max} = \left\{\alpha_1 + \alpha_2 \left(\frac{w_s}{u_*}\right)^{\alpha_3}\right\} u_* h, \quad \text{for} \quad \frac{z-z_b}{h} \geq 0.5 \tag{6}$$

in which: u_* = bed-shear velocity, $\alpha_1 = 0.1$, $\alpha_2 = 0.38$, $\alpha_3 = 4.31$ for flumes, $\alpha_1 = 0.13$, $\alpha_2 = 0.20$, $\alpha_3 = 2.12$ for natural channels (empirical) constants).

Non-uniform flow
In case of flow separation the maximum value of the diffusion coefficient ($\varepsilon_{s,\max}$) in the deceleration zone of the trench is assumed to be proportional to the thickness of the mixing layer and the velocity difference across the mixing layer. The measured local (mean) flow velocities and local shear stresses were used to determine the proportionality factor [3]. For steep-sided trenches the computed diffusion coefficient in the deceleration zone was found to be about twice as large as the value for equilibrium conditions, which seems reasonable considering the high turbulence level in this zone of the trench. In the relaxation and acceleration zone of the trench the diffusion coefficient is described by a set of empirical relations which give a gradual transition from the (high) value of the diffusion coefficient in the deceleration zone to the downstream value of the diffusion coefficient, described by equations (5) and (6). In all zones the

vertical distribution of the diffusion coefficient is parabolic in the lower half and constant in the upper half of the flow depth. If flow separation does not occur (relatively low turbulence level), the diffusion coefficient is described by the relations for uniform flow.

4. Boundary conditions

1. upstream boundary: the concentration profile c(o,z) must be given as a function of time
2. surface boundary: $(w_s\, c + \varepsilon_s \frac{\delta c}{\delta z})_{z = z_b + h} = 0$ (7)
3. bed boundary:

   $$\frac{\delta c(x, z_b + z_a)}{\delta z} = 0, \text{ in the sedimentation zone, where } \frac{\delta s}{\delta x} < 0 \qquad (8)$$

   $$c(x, z_b + z_a) = c_e, \text{ in the erosion zone, where } \frac{\delta s}{\delta x} \geq 0 \qquad (9)$$

   in which: s = local sediment transport capacity, z_a = small level above the bed, c_e = equilibrium bed-concentration.
   The equilibrium bed-concentration is computed from a transport formula using the concentration and the flow velocity distribution for uniform flow conditions [1], [2]. In case of flow separation and the presence of a recirculation zone, the bed boundary condition is applied at the dividing stream line (above which the discharge remains constant). The introduction of a new bed boundary is necessary because the diffusion-convection equation (3) cannot be solved for negative longitudinal flow velocities. The recirculation zone is treated as a "black box" in which the sediment concentrations are assumed to be constant in vertical direction and equal to the computed concentration at the dividing streamline.

4. the initial (t = 0) bed profile (z_b) must be given as a function of longitudinal distance.

5. the upstream discharge (q) and flow depth (h) must be given as a function of time (the water surface is supposed to be horizontal).

5. Solution method

Due to the large vertical concentration gradients near the bed, it is desirable to reduce the vertical grid size towards the bed which is done by means of the following transformation:

$$z' = \int_{z_b + z_a}^{z} \left(\frac{w_s}{\varepsilon_s}\right) dz \qquad (10)$$

To obtain a rectangular grid pattern for each configuration, a second transformation is applied:

$$z'' = \frac{z'}{z'_{max}} \qquad (11)$$

A six-point implicit, finite-difference sheme is used to solve the transformed diffusion-convection equation.

6. Verification

Laboratory conditions

Three tests with different dimensions of the trench were carried out [4]. The upstream flow velocity (= 0.5 m/s) and flow depth (= 0.4 m) were not varied. The sediment bed consisted of fine sand (D_{50} = 160 μm). At the upstream boundary sand was supplied at a rate of 0.040 kg/sm.

To determine the longitudinal and vertical distribution of the concentrations, water-sediment samples were collected simultaneously by means of a siphon sampler which consisted of a short hose run connected to an intake nozzle. From these measurements the suspended load transport was estimated to be about 0.030 (± 0.06) kg/sm. Consequently, the bed load transport was about 0.010 kg/sm. The size of the suspended sediment varied from 120 μm near the water surface to 150 μm naer the bed, which results in a representative particle fall velocity in the range 0.011-0.015 m/s for a water temperature of 15°C. To describe the local transport capacity of the flow in relation to the average flow velocity, a simple empirical formula was derived. The bed boundary condition was applied at a level of 0.0125 m above the bed, which is about half the ripple height.

To represent the measured upstream concentration profile correctly, the diffusion coefficient ($\varepsilon_{s, max}$) was varied using a constant particle fall velocity of 0.013 m/s. The best agreement was obtained for $\varepsilon_{s, max}$ = 0.00165 m^2/s, which seems to be a realistic value because equation (6) predicts a similar value.

Figure 1 presents measured and computed flow velocity and sediment concentration profiles for the steep-sided trench (1 : 3) at the start of the test (t = 0). In the deceleration zone (profiles 1, 2, 3, 4, 5) the agreement between measured and computed flow velocities is fairly good; in the relaxation and acceleration zone (profiles 6, 7, 8) relatively large deviations do occur, particularly near the bed. Comparison of measured and computed concentration profiles show large deviations in the deceleration zone (profiles 2, 3, 4, 5). Probably, the relative intensive mixing process in this zone is not represented correctly in the model by the (parabolic-constant) diffusion coefficient distribution. Also, the assumption of a zero bed-concentration gradient at the dividing stream line expressing no re-entrainment of sediment may be too crude. In the acceleration zone the agreement between measured and computed concentrations is also not optimal, probably due to the fact that the high flow velocities near the bed are not fully represented, while also the applied bed-concentration condition results in relatively low bed concentrations.

Figure 2 shows measured and computed bed level profiles at 7.5 and 15.0 hours for the steep-sided trench. The computed sedimentation level is remarkably good. The bed level in the erosion zone is not predicted correctly, probably because the applied bed boundary condition is not optimal. At the moment theoretical and experimental research is going on at the Delft Hydraulics Laboratory to improve the bed boundary condition.

To determine the influence of the main controlling parameters on the sedimentation process, being the upstream diffusion coefficient, the particle fall velocity, the upstream sediment transport and the bed boundary condition, a sensitivity analysis was carried out using the mathematical model.

Figures 3 and 4 show the influence of the particle fall velocity and the upstream diffusion coefficient for the trench with side slopes of 1 : 7.

For both parameters the influence is largest in the sedimentation zone, because the particle fall velocity and the diffusion coefficient modify the upstream concentration profile, while the particle fall velocity also affects the downward sediment transport in the trench. The longitudinal distribution of the

---

4. Delft Hydraulics Laboratory,
   Mathematical Model for the Computation of the Siltation in Dredged Trenches,
   Report R 1267-V/M 1570, 1980.

diffusion coefficient appeared to be of minor importance. The influence of the upstream sediment transport and the bed boundary condition will be discussed for the field experiments.

Field conditions

In the entrance of the Oosterschelde estuary, a large storm surge barrier will be build. The piers of the barrier will be founded in a dredged trench. To estimate the sedimentation in the trench, a test pit with a depth of 4.5 m, a bottom length of 200 m and side slopes of 1 : 6 (Figure 5) was dredged. The local flow depth was 21.5 m. Outside the trench the bed was protected by a sand tight matress. The neap-spring tidal cycle was represented by a design tide, which was schematized in 4 quasi-steady flow periods of 2 hours each. The composition of the bed material was determined from bed samples outside the testpit (D_{50} = 300 μm). From sieve analysis and settling experiments of suspended sediment samples the representative diameter of the suspended sediment was estimated to vary from 150-200 μm, which results in a particle fall velocity in the range 0.012-0.0185 m/s for a water temperature of 5^oC (winter period).

Sediment concentration [5], flow velocity measurements and bed form trackings were used to determine the suspended and bed load transport. The bed boundary condition was applied at a level of 0.125 m, which is about half the bed form height. To represent the measured upstream concentration profile correctly, the diffusion coefficient ($\varepsilon_{s,max}$) was varied using a constant particle fall velocity of 0.015 m/s.

Figure 5 shows computed and measured bed level profiles after 180 days. The computed bed level using equation (8) and (9) as bed-boundary condition shows remarkably good agreement.

To estimate the influence of the bed boundary condition, the sedimentation level was also computed using the equilibrium bed-concentration in stead of the zero bed-concentration gradient in the sedimentation zone (Figure 5). This alternative bed boundary condition results in significantly less sedimentation because the equilibrium bed-concentration condition leads to higher concentrations in the trench. Finally, the influence of the upstream sediment transport is discussed. As the amount of sedimentation in the trench is almost linear dependent on the value of the upstream sediment transport [2], it will be clear that a significantly different prediction is obtained if the upstream sediment transport is not estimated sufficiently accurate. Therefore, detailed and reliable field data over a long period are of essential importance.

---

5. Rijn, L.C. van,
   Pump-Filter Sampler, Design of an Instrument for Measuring Suspended Sand Concentrations in Tidal Conditions,
   Research Report S 404-I, Delft Hydraulics Laboratory, June 1979.

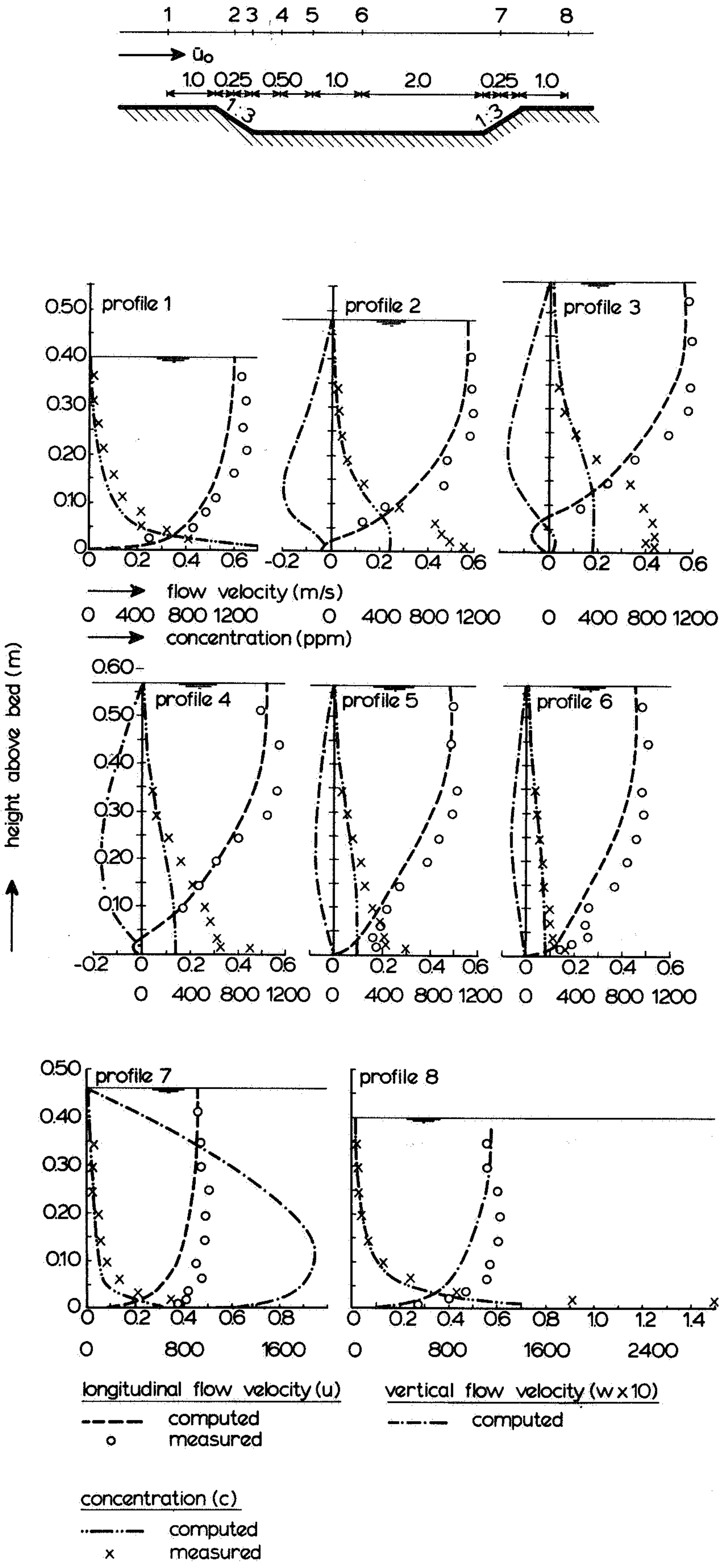
1 2 3 4 5 6 7 8
ū₀
1.0 0.25 0.50 1.0 2.0 0.25 1.0
1:3
1:3
profile 1
profile 2
profile 3
profile 4
profile 5
profile 6
profile 7
profile 8
height above bed (m)
flow velocity (m/s)
concentration (ppm)
0 0.10 0.20 0.30 0.40 0.50 0.60
-0.2 0 0.2 0.4 0.6
0 400 800 1200
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
0 800 1600 2400
longitudinal flow velocity (u)
computed
measured
vertical flow velocity (w x 10)
computed
concentration (c)
computed
measured

figure 1

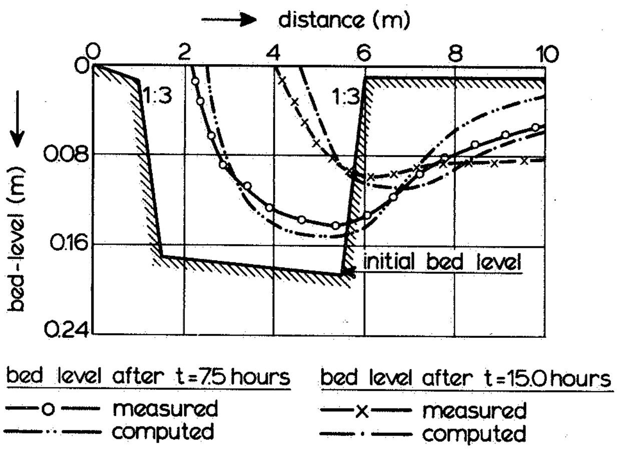

figure 2

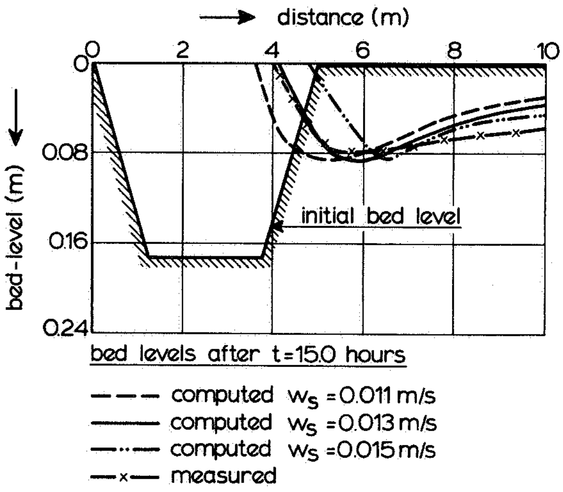

figure 3

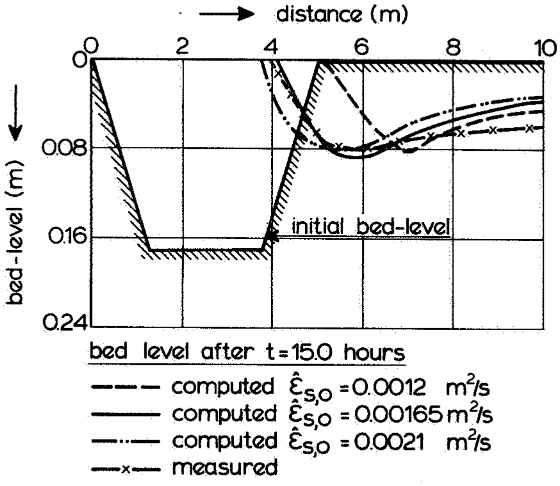

figure 4

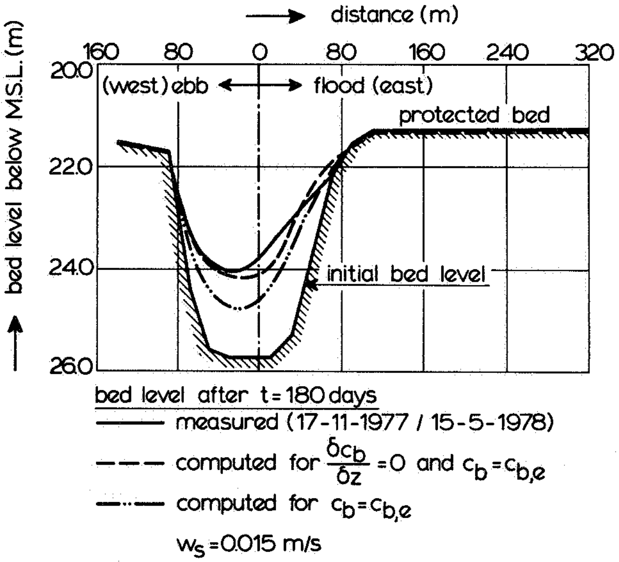

figure 5