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ON INTERNALLY GENERATED ESTUARINE TURBULENCE

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1 Introduction

The processes producing turbulence and mixing must be examined carefully when dealing with stably stratified fluids. According to Turner (1973) a distinction must be made between "external" turbulence generated directly at a solid boundary, and "internal" turbulence arising in the interior. This observation is also applicable to estuarine mixing. At the one extreme, in a well mixed estuary the turbulence is primarily boundary generated. At the other extreme, in a highly stratified estuary with an arrested salt wedge the turbulence is primarily generated at the interface, i.e. in the interior of the stratified fluid. Nevertheless, the distinction between external and internal turbulence has not been made in the literature on estuarine mixing as yet (Fischer, 1976). The significance of this distinction is the subject of the present paper.

The first part of the paper gives a theoretical criterion to determine under which circumstances the effect of internal turbulence on the production of turbulent energy exceeds the effect of external turbulence. The criterion is based on an approximate solution of the equation of motion in the longitudinal direction. The second part of the paper is an analysis of salinity intrusion field data collected in the Rotterdam Waterway and in the Chao Phya Estuary. This analysis confirms that above a certain level of stratification internal effects are to be taken into account. The third part of the paper elaborates upon the implications of the aforementioned findings for the numerical two-dimensional simulation (with longitudinal and vertical space dimensions) of salinity intrusion in partly mixed estuaries.

2 Derivation of criterion

Neglecting the variations of the velocity, salt concentration and water level in the lateral direction, the equation of motion in the longitudinal direction reads

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} + \frac{1}{\rho}(h-y)g\frac{\partial \bar{\bar{\rho}}}{\partial x} - \frac{1}{\rho}\frac{\partial \tau}{\partial y} = 0 \qquad (1)$$

(a) (b) (c) (d) (e) (f)

where

x : longitudinal coordinate measured from mouth of estuary; landward direction is positive x-direction,

y : vertical coordinate (y = 0 refers to bottom which is assumed to be horizontal),

t : time,

u : velocity component in longitudinal direction; landward direction is positive,

v : velocity component in vertical direction,

g : acceleration by gravity,

h : depth,

ρ : density,

$\bar{\bar{\rho}}$: density averaged over fraction of depth ranging from y = y tot y = h,

τ : turbulent shear (shear is positive when decelerating fluid above flowing in positive direction).

In Eq. (1) term (a) is replaced by its depth mean value. This approximation is the more justified the closer u approaches its extreme values ($\partial u/\partial t = 0$). In sufficiently stratified estuaries with cross-sections varying gradually with x terms (b) and (c) may be neglected in comparison with term (e), as shown by Abbott (1960). Making these approximations, and taking into account that

$$\tau = \tau_b \quad \text{for} \quad y = 0 \quad ; \quad \tau = 0 \quad \text{for} \quad y = h \qquad (2)$$

where

τ_b: bottom shear,

integration of Eq. (1) with respect to y gives

$$\tau_b = -\rho\, h\frac{\partial \bar{u}}{\partial t} - \rho\, gh\frac{\partial h}{\partial x} - g\int_o^h (h-y)\frac{\partial \bar{\bar{\rho}}}{\partial x}\,dy \qquad (3)$$

and

$$\tau = \tau_b\frac{(h-y)}{h} + g\frac{(h-y)}{h}\int_o^h (h-y)\frac{\partial \bar{\bar{\rho}}}{\partial x}\,dy - g\int_y^h (h-y)\frac{\partial \bar{\bar{\rho}}}{\partial x}\,dy \qquad (4)$$

where

$\bar{u}$: depth mean value of u.

Eq. (4) separates τ into two fractions, proportional to τ_b and $\partial\bar{\bar{\rho}}/\partial x$ respectively, and may be written as

$$\tau = \tau_{ex} + \tau_{in} \qquad (5)$$

with

$$\tau_{ex} = \tau_b\frac{(h-y)}{h} \quad ; \quad \tau_{in} = g\frac{(h-y)}{h}\int_o^h (h-y)\frac{\partial \bar{\bar{\rho}}}{\partial x}\,dy - g\int_y^h (h-y)\frac{\partial \bar{\bar{\rho}}}{\partial x}\,dy \qquad (6)$$

The fraction τ_{ex} is equal to τ_b for $y = 0$ and decreases linearly with increasing y. The fraction τ_{in} is proportional to $\partial\bar{\bar{\rho}}/\partial x$ and is equal to zero both for $y = 0$ and $y = h$. Consequently τ_{ex} and τ_{in} may be looked upon as:

τ_{ex} : contribution of externally generated turbulence to turbulent shear, and
τ_{in} : same of internally generated turbulence.

For a well mixed estuary ($\bar{\bar{\rho}} = \bar{\rho}$, where a single overbar denotes a depth mean value) Eq. (6) gives

$$\frac{\tau_{in}}{\tau_{ex}} = \frac{1}{2}\,\frac{gh^2\,\frac{\partial\bar{\rho}}{\partial x}}{\tau_b}\,\frac{y}{h} \qquad \text{(well mixed estuary)} \tag{7}$$

For a highly stratified estuary ($\partial\bar{\rho}/\partial x$ due to slope of sharp interface between upper layer and lower layer, each having constant density) Eq. (6) gives

$$\frac{\tau_{in}}{\tau_{ex}} = \frac{gh^2\,\frac{\partial\bar{\rho}}{\partial x}}{\tau_b}\,\frac{y_i}{h} \qquad \text{(highly stratified estuary)} \tag{8}$$

where
y_i : value of y corresponding to interface.

For a partly mixed estuary, the zone with the greatest vertical density gradient being located at y_i, the value of τ_{in}/τ_{ex} will be between the values given by Eq. (7) (after substituting $y = y_i$) and Eq. (8).

The sign of τ_b changes during a tidal cycle, depending on the direction of $\bar{u}$. The sign of $\partial\bar{\rho}/\partial x$ remains the same during a tidal cycle. Consequently Eqs. (7) and (8) give positive values of τ_{in}/τ_{ex} during ebb and negative values during flood.

In accordance with Eq. (5)

$$P = (\tau_{ex}+\tau_{in})\,\frac{\partial u}{\partial y} \tag{9}$$

where
P : production of turbulent energy.

Though the magnitude of $\partial u/\partial y$ is affected by density effects, Eq. (9) shows that P is equally influenced by external and internal effects for

$$\left|\frac{\tau_{in}}{\tau_{ex}}\right| = 1 \tag{10}$$

Internal and external effects are equally important for $\tau_{in}/\tau_{ex} = -1$, as then the production of turbulent energy is zero.

Eq. (10) provides the criterion to distinguish conditions with production of turbulent energy being influenced primarily by external or internal effects. It has to be applied in conjunction with Eqs. (7) and (8). It does not require information on turbulence properties as such, as aimed for in its derivation.

With application of the criterion τ_b has to be known. This quantity has been derived from Eq. (3), assuming that

$$\frac{\bar{\rho}\, g |\bar{u}| \bar{u}}{C^2_{hom}} \approx -\rho\, h \frac{\partial \bar{u}}{\partial t} - \rho\, g \frac{\partial h}{\partial x} \qquad (11)$$

where

C_{hom} : Chézy coefficient pertaining under homogeneous conditions.

This assumption implies that the integral of Eq. (3) has little effect on the sum of the terms at the right hand side of Eq. (11) for $\bar{u}$ having the same value.

Defining the Chézy coefficient as

$$\frac{\bar{\rho}\, g |\bar{u}| \bar{u}}{C^2} = \tau_b \qquad (12)$$

where

C : Chézy coefficient.

Eqs. (3) and (11) give

$$\frac{\bar{\rho}\, g |\bar{u}| \bar{u}}{C^2} = \frac{\bar{\rho}\, g |\bar{u}| \bar{u}}{C^2_{hom}} - g \int_0^h (h-y) \frac{d\bar{\bar{\rho}}}{dx}\, dy \qquad (13)$$

With landward velocities both terms at the right hand side have the same sign. With seaward velocities these terms have opposite signs. Consequently with seaward velocities the Chézy coefficient has greater values than with landward velocities.

3 Analysis of field data

The analysis of field data has been made on the basis of data collected in the partly mixed Rotterdam Waterway available for 1908 and 1956 (derived from Harleman and Abraham, 1966) and for 1971 (presented by Rijkswaterstaat, 1971). In addition, the analysis has been made for field data under well mixed conditions collected in the Chao Phya Estuary available for 1962 (presented by Nedeco, 1963).

Figure 1 shows chlorinity distributions as observed in the Rotterdam Waterway under the 1971 conditions, the most stratified ones included in the analysis. Figure 2 shows well mixed density distributions as observed in the Chao Phya Estuary.

The objective of the analysis is to illustrate that internal effects may not be neglected in partly mixed estuaries. Therefore the value of τ_{in}/τ_{ex} which has to be substituted into Eq. (10) has been derived from Eq. (7), which gives the correct value of τ_{in}/τ_{ex} for well mixed conditions and which gives a lower limit of τ_{in}/τ_{ex}

for partly mixed conditions. Thus, Eq. (7) does not lead to an overestimation of the importance of internal effects.

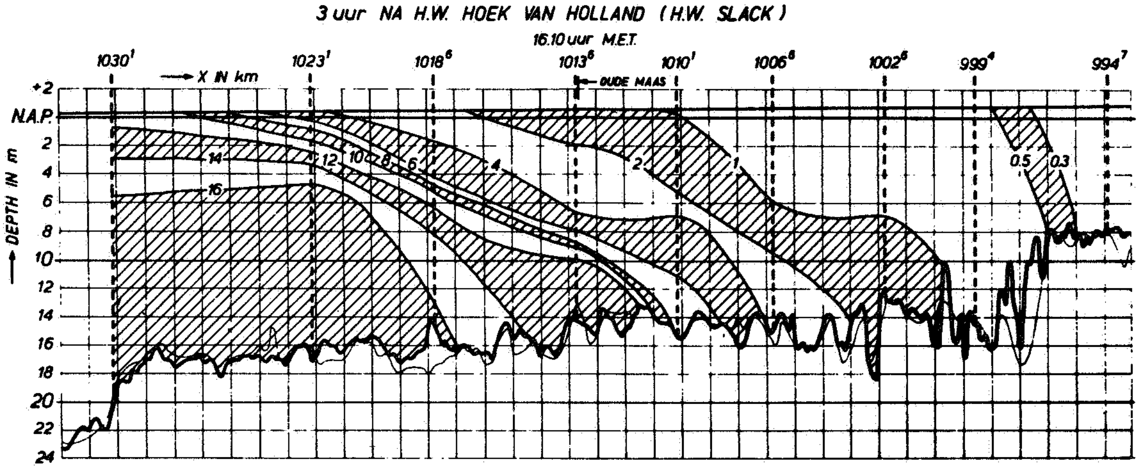

Figure 1 Rotterdam Waterway, 1971 (E = 0.19), chlorinity data (Rijkswaterstaat, 1971); numbers refer to chlorinity in gr/l

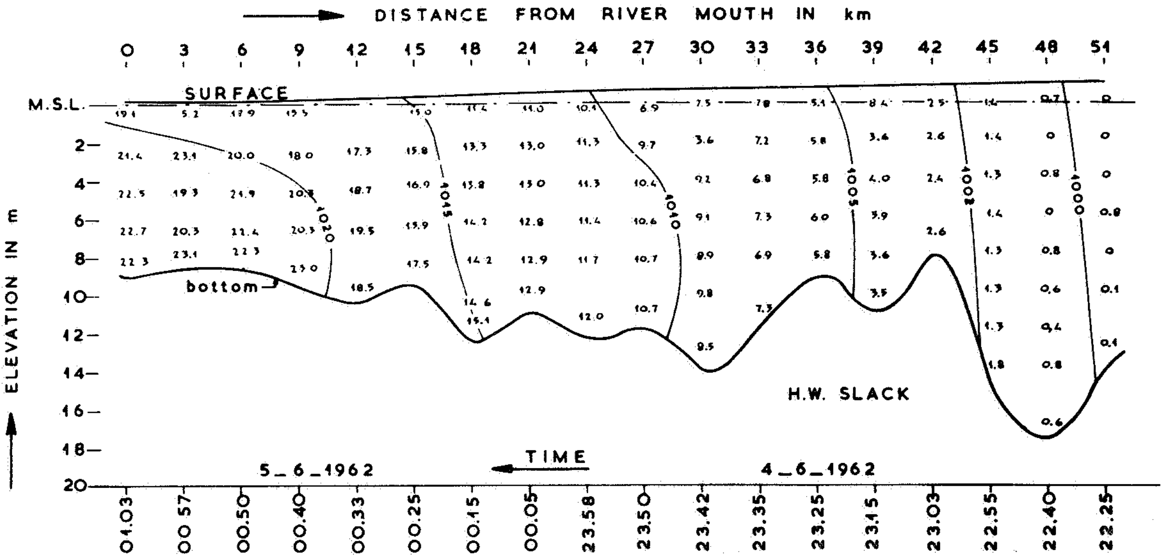

Figure 2 Chao Phya Estuary, 1962 (E = 34.8), density distributions (Nedeco, 1963); numbers refer to density in kg/m^3

With application of Eq. (7), the value of y must be specified. In well mixed estuaries τ_{in} has its maximum value at $y = \frac{1}{2}h$. This makes it reasonable to select $y = \frac{1}{2}h$ as the level of interest. In estuaries with some degree of stratification it is reasonable to take the level with the greatest vertical density gradient as the level of interest, as the eddy diffusivity at this level governs the degree of stratification of the estuary. This level varies with the distance from the mouth of the estuary. In the analysis this level is arbitrarily taken at $y = \frac{1}{2}h$.

Table 1 summarizes the analysis, giving the following characteristics of the considered estuaries:

- characteristics of the considered section of the estuary: depth (h), cross section (A), fresh water flow rate (Q_{fr}), C_{hom},
- characteristics of the flow through the mouth of the estuary: maximum velocity in the flood direction ($\bar{u}_{m.f}$), same in the ebb direction ($\bar{u}_{m.e}$), tidal prism (P_t). i.e. the volume of sea water entering the estuary on the flood tide,
- salinity parameters: density gradient during flood ($\partial\bar{\rho}/\partial x$, flood), same during ebb ($\partial\bar{\rho}/\partial x$, ebb), estuary number (E), value of τ_{in}/τ_{ex} when velocity has maximum value of the flood direction ($(\tau_{in}/\tau_{ex})_{m.f}$), same when velocity has maximum value in the ebb direction ($(\tau_{in}/\tau_{ex})_{m.e}$), fraction of flood period with $|\tau_{in}/\tau_{ex}|$ exceeding one (α_{flood}), fraction of ebb period with τ_{in}/τ_{ex} exceeding one (α_{ebb}).

For the Rotterdam Waterway the magnitude of C_{hom} is taken to be equal to 70 $m^{\frac{1}{2}}/s$. For the 1956 conditions substituting this value into Eq. (13) gives values of C for the flood tide and the ebb tide which agree favourably with values derived from field measurements (Dronkers, 1969). The 1956 ebb value is also applied as the 1971 ebb value. For the Chao Phya Estuary a value of C_{hom} equal to 80 $m^{\frac{1}{2}}/s$ has been adapted. This value is given for the homogeneous part of the estuary by Nedeco (1963).

The degree of stratification of the estuaries under consideration is expressed in terms of the estuary number, E, as defined by Harleman and Thatcher (1974). Decreasing values of E indicate an increasing level of stratification, This can be seen by comparing the vertical salinity distribution found in the Rotterdam Waterway 1971 (Fig. 1, E = 0.19) and in the Chao Phya Estuary (Fig. 2, E = 34.8).

In accordance with quantitative considerations given by Abbott (1960) terms (b) and (c) of Eq. (1) may be neglected in comparison with term (e) for the Rotterdam Waterway conditions listed in Table 1, and probably not for the Chao Phya Estuary conditions listed in this table.

The parameter values listed in Table 1 indicate that for the Rotterdam Waterway 1971 conditions internally generated turbulence is important: during the whole ebb tide $\tau_{in}/\tau_{ex} > \frac{1}{2}$, and during half of the ebb tide $\tau_{in}/\tau_{ex} > 1$; during the whole flood tide $|\tau_{in}/\tau_{ex}| > 1/5$, and during 1/3 of the flood tide $|\tau_{in}/\tau_{ex}| > 1$. In addition Table 1 shows that internally generated turbulence may not be neglected in the Rotterdam Waterway 1956 conditions: during the whole ebb tide $\tau_{in}/\tau_{ex} > 1/4$ and during 1/3 of the ebb tide $\tau_{in}/\tau_{ex} > 1$; during the whole flood tide $|\tau_{in}/\tau_{ex}| > 1/6$, and during 1/4 of the flood tide $|\tau_{in}/\tau_{ex}| > 1$.

The parameter values given for the Chao Phya Estuary under well mixed conditions indicate that internally generated turbulence is important only around tidal slack.

Summarizing, both externally and internally generated turbulence are to be taken into account for estuaries with the degree of stratification as found in the Rotterdam Waterway 1971 and 1956 conditions ($E = 0.19$ and $E = 0.59$). In well mixed estuaries internally generated turbulence is important only around tidal slack.

The parameter values of Table 1 were derived replacing $\partial u/\partial t$ by its depth mean value in Eq. (1). This approximation is not justified around tidal slack. Nevertheless it seems justified to make the aforementioned conclusions as around tidal slack bottom shear has small values, implying that externally generated turbulence is relatively unimportant.

4 Implications for estuarine modelling

Under stratified conditions

$$\frac{\tau}{\rho} = L^2 \frac{\partial \bar{u}}{\partial y} \left|\frac{\partial \bar{u}}{\partial y}\right| F(Ri) \quad ; \quad \frac{\tau}{\rho} = L \; e^{\frac{1}{2}} \frac{\partial \bar{u}}{\partial y} f(Ri) \qquad (15)$$

(a) (b)

where

L : length scale of turbulence,

e : kinetic energy of turbulence,

Ri : gradient Richardson number, defined as

$$Ri = \frac{-g \frac{\partial \rho}{\partial y}}{\rho \left(\frac{\partial u}{\partial y}\right)^2} \qquad (16)$$

f(Ri), F(Ri) : damping functions.

By definition the damping functions satisfy

$$\frac{d\ f(Ri)}{d\ Ri} \leq 0 \quad ; \quad \frac{d\ F(Ri)}{d\ Ri} \leq 0 \qquad (17)$$

The length scale L may be rather problem dependent. The effect of stratification is not clear as yet (see e.g. a literature survey presented by Vreugdenhil, 1974).

When the turbulence is exclusively generated at the solid boundary

$$L = L_{ex} = f(y,h) \qquad (18)$$

where

L_{ex} : length scale of externally generated turbulence.

In two-dimensional numerical simulation of salinity intrusion it is common practice to take Eq. (15) as a starting point, deriving L from Eq. (18). Expression (a) of Eq. (15) and Eq. (18) provide the basis for the two-dimensional simulations by Odd and Rodger (1978) and Perrels and Karelse (1977 and 1980). Expression (b) of Eq. (15) and Eq. (18) are applied in the simulations of Smith and Dyer (1978) and Liu and Leendertse (1978).

Liu and Leendertse (1978) express the damping in terms of a modified Richardson number, which is obtained by eliminating $\partial u/\partial y$ out of Eq. (16) assuming

$$\frac{\partial u}{\partial y} :: \frac{e^{\frac{1}{2}}}{L_{ex}} \tag{19}$$

The length scale Δy of Eq. (19) which in the limit $\Delta y \rightarrow 0$ represents the length scale ∂y decreases with increasing stratification, while the length scale L_{ex} does not. Hence the physical meaning of the modified Richardson number does not coincide with that of the actual Richardson number. This makes it uncertain whether it is correct to express the damping in terms of the modified Richardson number.

The length scale and velocity scale of purely internal turbulence are different from the ones for purely external turbulence. Thus, when both internal and external turbulence are to be taken into account, the length scale and the velocity scale of the resulting turbulence vary with the ratio τ_{in}/τ_{ex}. Hence, for values of the ratio $|\tau_{in}/\tau_{ex}|$ sufficiently large for internal turbulence to be important the magnitude of L deviates from the value given by Eq. (18), which pertains for values of $\tau_{in}/\tau_{ex} \approx 0$. The aforementioned simulations adhere to Eq. (18) and consequently to a value of L which does not vary with time. In reality τ_{in}/τ_{ex} and consequently L vary with time during a tidal cycle. This can only be compensated for by accepting the damping functions f(Ri) and F(Ri) to vary with time during the tidal cycle. Nevertheless the aforementioned simulations are based on both Eq. (18) and apply the same damping functions throughout the tidal cycle. This makes it unlikely that these simulations can reproduce the salinity distribution in all its details.

Perrels and Karelse (1980) compare their computational results with detailed experimental data obtained in the Delft Hydraulics Laboratory salinity flume (Van Rees and Rigter, 1969). They found systematic differences between the measurements and the computational results, which could be due to the use of the same damping function throughout the tidal cycle.

Summarizing application of Eq. (15) in conjunction with Eq. (18) means neglecting the effect of stratification on the length scale of turbulence. This has to be compensated for by allowing the damping function of Eq. (15) to vary with time. This applies to estuaries with values of $|\tau_{in}/\tau_{ex}|$ sufficiently large for internal turbulence to be important.

Table 1 Field data and salinity parameters Rotterdam Waterway and Chao Phya Estuary

	Rotterdam Waterway			Chao Phya Estuary	
	7th April 1971	26th June 1956	22nd July 1908	4th and 5th June 1962	
				talweg	average depth
Field data	(km 1014-1030)	(km 1014-1030)	(km 1014-1030)	(km 15-36)	(km 15-36)
h (m)	$15.8^{*)}$	$13.0^{*)}$	$7.7^{*)}$	12	8
A (m^2)	6500	5300	3600	-	3200
Q_f (m^3/s)	1550	960	430	90	90
C_{hom} ($m^{\frac{1}{2}}/s$)	70	70	70	80	80
Field data mouth	(km 1030)	(km 1030)	(km 1030)	(km 2)	(km 2)
$\bar{u}_{m.f}$ (m/s)	1.05	1.1	1.2	1.3	1.3
$\bar{u}_{m.e}$ (m/s)	1.05	1.1	1.2	1.3	1.3
P_t	$51\ 10^6$	$66\ 10^6$	$46\ 10^6$	10^8	10^8
Salinity parameters					
$\partial\bar{\rho}/\partial x$ (flood) $(\frac{kg/m^3}{m})$	$1.25\ 10^{-3}$	$1.4\ 10^{-3}$	$1.8\ 10^{-3}$	$0.6\ 10^{-3}$	$0.6\ 10^{-3}$
	(km 1023-1015)	(km 1023-1015)	(km 1025-1017)	(km 15-36)	(km 15-36)
$\partial\bar{\rho}/\partial x$ (ebb) $(\frac{kg/m^3}{m})$	$1.2\ 10^{-3}$	$1.1\ 10^{-3}$	$1.25\ 10^{-3}$	$0.3\ 10^{-3}$	$0.3\ 10^{-3}$
	(km 1030-1021)	(km 1030-1023)	(km 1030-1025)	(km 15-30)	(km 15-30)
$y = \frac{1}{2}h$ (m)	7.9	6.5	3.8	6	4
E (Eq. 14)	0.19	0.56	1.9	17.2	34.8
$(\tau_{in}/\tau_{ex})_{m.f}$ (Eq. 7)	-0.21	-0.16	-0.08	-0.08	-0.04
$\alpha_{flood}^{**)}$	0.30	0.26	0.18	0.18	0.13
$(\tau_{in}/\tau_{ex})_{m.e}$ (Eq. 7)	0.47	0.26	0.07	0.05	0.02
$\alpha_{ebb}^{***)}$	0.48	0.34	0.17	0.14	0.09

*) depth of talweg about equal to average depth

**) α_{flood}: fraction of flood tide with $|\tau_{in}/\tau_{ex}| > 1$

***) α_{ebb} : fraction of ebb period with $\tau_{in}/\tau_{ex} > 1$

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