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numerical aspects of bed level predictions for alluvial river bends

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Synopsis

Tentative calculations have been executed to predict the bed level in a river bend. Two-dimensional flow models have been applied to describe the depth-averaged velocities. The bed level variation is described by a continuity equation and an adapted Meyer-Peter and Muller sediment transport formula together with a formula for the direction of the sediment transport. The adaptation involves gravity effects and the effect of spiral motion. These effects are also included in the direction of sediment transport. It is shown that the mathematical character of the system of equations describing bed-level fluid flow interaction depends greatly on the chosen sediment transport relation. The relation used allows an approximate description of the bed level by an advection-diffusion equation. Rather simple flow models predict flow fields with a poor correspondence to measured fields. The mutual differences are small for flat beds but large for uneven beds. In the case of axisymmetry a good agreement with respect to measurements for the equilibrium bed level is obtained. For a 180^{o} bend with straight inlet and outlet sections the order of magnitude of the bed level variation is correctly obtained by the mathematical model, but the asymmetry of the bed topography is not reproduced. The main reason for this deviation may the limited description of the spiral motion.

Introduction

Numerical methods to predict bed level variations for alluvial river or canal systems are well developed nowadays as far as one-dimensional descriptions are concerned. However, for a number of engineering problems (Jansen, 1979) such as bank protection, navigability, etc. a more detailed knowledge of the bed level behaviour is required. The present paper describes an investigation to understand this behaviour for river bends, and the development of a numerical method for predicting the 2-D bed topography and its time dependent development. The interaction between the flow field and the bed topography was included in the model. The present applications are restricted to bends with uniform width, fixed vertical side-walls, and sediment that can be characterized by one particle diameter only. Lag due to suspended sediment transport will not be considered. The equation will be presented in a rectangular coordinate system (x,y), but they have been solved in a system fitted to the geometry by polar coordinates. The present investigations were carried out within a research programme of the Dutch Government Public Works Department on rivers, in which the Delft University of Technology and the Delft Hydraulics Laboratory are participating.

Description of flow models

In a flow through a river bend a spiral motion occurs due to the local imbalance

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JANSEN, P. Ph.: Principles of river engineering, the non-tidal alluvial river, Pitman Publishing Ltd, London, 1979.

between the centrifugal forces and the hydrostatic pressure forces. Therefore the flow will be essentially three-dimensional. However, for the present application the use of a 3-D flow model will lead to exceptionally high computer costs. So attention has been directed to depth-averaged flow models. If this approach is not sufficiently accurate, applications involving no flow separation or moderate curvatures, a parabolic approximation to the 3-D problem may be acceptable. In literature much attention is paid to the flow field in a bend (Shukry, 1949; Leschziner and Rodi, 1978). The sediment transport is greatly influenced by the spiral motion of the flow, so when using depth-averaged flow equations, special adaptations are required to account for this effect. Later on attention will be paid to this point. The full expression for the depth-averaged equations are still complicated. In fact it can be questioned whether an accurate flow computation will be needed to determine the bed configuration only. To investigate this question three flow models have been applied. The first model (I) is described by the equations:

$$\frac{\partial uh}{\partial x} + \frac{\partial vh}{\partial y} = 0, \qquad (1)$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0, \qquad (2)$$

where (u,v) is the depth-averaged horizontal velocity, h the waterdepth and (x,y) the horizontal space coordinates. In this case it is assumed that the convective terms and the pressure terms are the leading order terms of the depth-averaged equations. The second model (II) consists of equation (1) and the equation:

$$\frac{\partial}{\partial x}\left(\frac{v\sqrt{u^2+v^2}}{C^2 h}\right) - \frac{\partial}{\partial y}\left(\frac{u\sqrt{u^2+v^2}}{C^2 h}\right) = 0, \qquad (3)$$

where C is Chézy's roughness coefficient. For this model it is assumed that the pressure and the bottom friction terms are the dominating terms.
Using a stream function Ψ both flow models can be represented by:

$$\frac{\partial}{\partial x}\left(K\frac{\partial \Psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial \Psi}{\partial y}\right) = 0\ , \qquad (4)$$

in which $K = h^{-1}$ for flow model I and $K = C^{-2}h^{-3}\sqrt{(\frac{\partial\Psi}{\partial x})^2+(\frac{\partial\Psi}{\partial x})^2}$ for flow model II

Finally an adapted version of the analytical solution derived by Engelund (1974) has been applied (Model III).
In the above formulas the time-derivatives of the velocities are not included.

ENGELUND, F.: Flow and bed topography in channel bends, Journal of the Hydraulics Division, ASCE, Vol. 100, Hy 11, Nov. 1974.
SHUKRY, A.: Flow around bends in an open flume, Transactions ASCE, June 1949, paper no. 2411; LESCHINER, M. and RODI, W.: Calculation of Three-dimensional turbulent flow in strongly curved open channels, Report SFB 80/T/126, University of Karlsruhe, May 1978.

It is assumed that the change of the flow field will be forced by bed level variations only, and since the characteristic time scales of the fluid flow will be small with respect to the time scales of bottom changes these terms can be neglected (quasi-stationary approach).

Description of bed level changes

The changes of the bed level are described by a continuity equation:

$$\frac{\partial z_b}{\partial t} + \frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} = 0, \qquad (5)$$

where z_b is the bed level and S_x, S_y are the sediment transport rates in x- and y-directions respectively.

The use of one of the existing sediment transport formulas is rather doubtful because these empirical formulas, lacking a clear physical composition, have been calibrated only for straight uniform flow with "flat" beds and include averaged dune and ripple effects. For the present application the effect of the local bed gradients on the sediment transport has to be included. Assuming that the Meyer-Peter and Muller formula relates the sediment transport rate to the total of forces along the bed exerted on the grains, adaptations are derived to account for the gravity and the spiral motion (Koch, 1980):

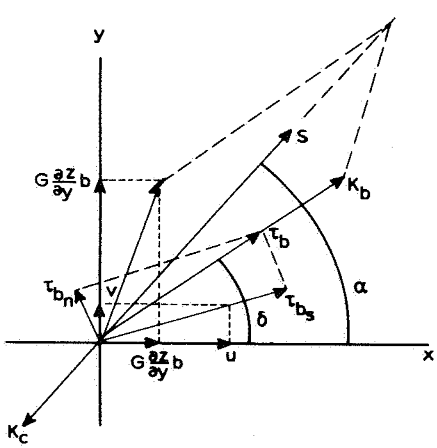

Figure 1 Definition sketch

$$S = a\,(A\tau_b - B\tau_c)^{1.5} \qquad (6)$$

$$A = \cos\,(\alpha-\delta)$$

$$B = 1 + \left(\frac{\partial z_b}{\partial x}\cos\alpha + \frac{\partial z_b}{\partial y}\sin\alpha\right)/\tan\phi$$

where α is the direction of the resulting transport S, δ the direction of the bottom shear stress τ_b, and a en τ are constants depending on sediment properties. The direction is given by

$$\tan\alpha = \left(\sin\delta - D\frac{\partial z_b}{\partial x}\right)\left(\cos\delta - D\frac{\partial z_b}{\partial x}\right)^{-1} \qquad (7)$$

where D is a function of the bed shear stress and the sediment properties. Normally the bed shear stress is assumed tangent to the stream lines. In case of a spiral motion the direction of the bed shear stress deviates from the flow direction. This is indicated by:

$$\delta = \arctan\,(v/u) + \arctan\,(\tau_{bn}/\tau_{bs}) \qquad (8)$$

KOCH, F.G.: Bed level computations for axisymmetric curved channels, Delft Hydraulics Laboratory, Report R 657 - IX, 1980.

in which τ_{bs}, τ_{bn} are bed shear stress components tangent and perpendicular to the mean flow. The second term on the righthand side of (8) contains the only explicit influence of the spiral motion present in the used model. The following expressions are used for τ_{bs} and τ_{bn}:

$$\tau_{bs} = \rho \frac{g}{C^2} u^2, \qquad (9)$$

$$\tau_{bn} = -\rho \frac{2g}{\kappa C^2} \frac{h}{R_s} u^2 \left(1 - \frac{\sqrt{g}}{\kappa C}\right), \quad (\kappa = 0.41), \qquad (10)$$

in which g is the acceleration of gravity and R_s the radius of curvature of the local stream line. The expressions (9) and (10) are based on a similarity solution for the vertical distribution of the horizontal velocities (de Vriend, 1976). This certainly will not be correct near the walls. In fact it may be applied only to bends with a small depth/width ratio and moderate curvature. Also the inertia effect of the spiral motion is not included.

Boundary conditions

Due to the complex structure of the final sediment transport relation it is not quite clear which boundary conditions have to be used for the system of equations (1), (2) and (5) or (1), (3) and (5). Investigation of possible characteristic surfaces is therefore required. Assuming the general form

$$a_1 \frac{\partial z_b}{\partial x} + a_2 \frac{\partial u}{\partial x} + a_3 \frac{\partial v}{\partial x} + b_1 \frac{\partial z_b}{\partial y} + b_2 \frac{\partial u}{\partial y} + b_3 \frac{\partial u}{\partial y} = c \qquad (11)$$

for the equations (2) and (3) the characteristic surfaces ϕ are described by the following equations

$$b_2 \left(\frac{\partial \phi}{\partial y}\right)^2 + (a_2 - b_3) \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} + a_3 \left(\frac{\partial \phi}{\partial x}\right)^2 = 0 \qquad (12)$$

$$\left(\frac{\partial \phi}{\partial x}\right)^2 \frac{\partial S_x}{\partial p} + \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y}\left(\frac{\partial S_y}{\partial p} + \frac{\partial S_x}{\partial q}\right) + \left(\frac{\partial \phi}{\partial y}\right)^2 \frac{\partial S_y}{\partial q} = 0 \ , \qquad (13)$$

where p and q means the x- and y-derivative of z_b respectively. The first equation is determined by the flow model. Substitution of actual values for a_2, a_3, b_2 and b_3 demonstrates that this equation does not admit characteristic surfaces, as could be expected from the stream function equation (4), which equation actually will be solved. The stream function will be prescribed along all boundaries. At the outflow boundary a weaker condition could be imposed, but for the present test cases this influence was negligible.

The equation (13) only depends on the sediment transport formula. Suppose the general formula of the resulting sediment transport reads

$$S = a\,(K - K_c)^{1.5} , \qquad (14)$$

VRIEND, H.J. de: A mathematical model of steady flow in curved open channels, Delft University of Technology, Department of Civil Eng., Report no. 76-1, 1976

where K is the force exerted on a sediment particle by the fluid flow and the gravity and K_c the friction force between particle and bed (Shield criterion). The force K can be specified as:

$$K = \sqrt{F_x^2 + F_y^2} \tag{15}$$

$$F_x(\omega,\varepsilon) = \omega K_b \cos\delta - \varepsilon G \frac{\partial z_b}{\partial x}, \tag{16}$$

$$F_y(\omega,\varepsilon) = \omega K_b \sin\delta - \varepsilon G \frac{\partial z_b}{\partial y}. \tag{17}$$

The two contributions to F_x and F_y represent the flow force (first term), supposed to be independent of the bed gradients, and the gravity force. By the ω,ε coefficients the relative importance of the contributions can be changed. The direction of the transport is equal to the direction of the resulting force:

$$\tan\alpha = \frac{F_y(\omega_o,\varepsilon_o)}{F_x(\omega_o,\varepsilon_o)}. \tag{18}$$

Now some cases will be considered in which the magnitude of the sediment transport changes only slightly and the direction α does not change. However the character of the equations changes essentially.

- Suppose the total sediment transport rates will not be influenced by the gravity, but the directions will ($\omega=\omega_o=\varepsilon_o=1$, $\varepsilon=0$). Then expression (13) changes to

  $$\{\frac{\partial\phi}{\partial x} F_y(1,1) - \frac{\partial\phi}{\partial y} F_x(1,1)\}^2 = 0 \tag{19}$$

  So there are two characteristic surfaces.
- Suppose the influence of the gravity will be included in both the sediment transport rates and the directions ($\omega=\omega_o=\varepsilon=\varepsilon_o=1$). Equation (13) will only allow trivial solutions when:

  $$(\frac{\partial S_y}{\partial p} + \frac{\partial S}{\partial q})^2 - 4\frac{\partial S_x}{\partial p}\frac{\partial S_y}{\partial q} < 0. \tag{20}$$

  For this case the left side of (20) becomes:

  $$-G^2 \frac{S}{K}\frac{\partial S}{\partial K}. \tag{21}$$

  When sediment transport occurs (21) will always be negative. So in this case characteristics do not exist.
- Sediment will be transported even when the time averaged force (turbulent time scales) acting on the grains is less than the friction force K_c, because the instantanous force may exceed this force during certain time intervals. This is accounted for by taking $\omega > 1$. If the direction α is still based on the time-averaged force, the case $\omega > 1$, $\varepsilon=\omega_o=\varepsilon_o=1$ occurs. Then relation (20) becomes

  $$\frac{3}{2}K(1,1)\sin^2\{\alpha(\omega,1)-\alpha(1,1)\} - 4(K(\omega,1)-K_c)\cos\{\alpha(\omega,1)-\alpha(1,1)\}<0 \tag{22}$$

  In this case situations can be indicated of no, coinciding, or two different characteristics, depending on the actual flow velocities. So in a flow region, regions can be distinguished in which the equations have a different character. The physical significance (if any) of these regions is unclear. These examples demonstrate the large impact of apparently small changes in the sediment transport relations on the character of the equations and consequently on the solution method to be used.

The relation presently used corresponds to the second example. The chosen boundary conditions for this case are no transfer of sediment through the rigid sidewalls, a prescribed bed level at the inflow boundary and a prescribed gradient of the bed at the outflow boundary.

The solution procedure

The set of equations is solved by the use of finite differences. Equation (4) is solved using a 5 point scheme by a direct method, assuming the bed level to be fixed:

$$\{K_{i+\frac{1}{2},j}(\Psi_{i+1,j}-\Psi_{i,j})-K_{i-\frac{1}{2},j}(\Psi_{i,j}-\Psi_{i-1,j})\}/(\Delta x)^2+$$
$$+\{K_{i,j+\frac{1}{2}}(\Psi_{i,j+1}-\Psi_{i,j})-K_{i,j-\frac{1}{2}}(\Psi_{i,j}-\Psi_{i,j-1})\}/(\Delta y)^2=0 \qquad (23)$$

The equation (7) is discretized according to the FTCS (forward time steps, central space steps) approach. The sediment transport rates will be calculated between the grid points to avoid spatial oscillations:

$$\frac{z_{i,j}^{l+1}-z_{i,j}^{l}}{\Delta t}+\frac{S_{xi+\frac{1}{2},j}-S_{xi-\frac{1}{2},j}}{\Delta x}+\frac{S_{yi,j+\frac{1}{2}}-S_{yi,j-\frac{1}{2}}}{\Delta y}=0. \qquad (24)$$

When the velocity distribution is known S_x and S_y can be calculated and by (24) a new bed level is obtained. At the fixed walls a one-sided scheme is chosen which saves the overall conservative property of the system with respect to sediment transport. By alternating the calculations of the flow distribution and of the bed level, the interaction is obtained between the flow and the bed level. The process starts with a given or estimated bed level. To save computation time the calculation of the bed is executed n times successively before the flow field calculation is repeated. In these successive bed level steps uh and vh are kept constant. The magnitude of n is restricted. Its analysis will not be given here. Stability conditions for the used time step can be obtained by approximating equation (5) by an advection-diffusion equation:

$$\frac{\partial z_b}{\partial t}+c_x\frac{\partial z_b}{\partial x}+c_y\frac{\partial z_b}{\partial y}-\varepsilon_{xx}\frac{\partial^2 z_b}{\partial x^2}-\varepsilon_{xy}\frac{\partial^2 z_b}{\partial x\partial y}-\varepsilon_{yy}\frac{\partial^2 z_b}{\partial y^2}=0 \qquad (25)$$

Assuming uh or vh to be constant, expressions can be derived for c_x, c_y, ε_{xx}, ε_{xy} and ε_{yy}. The stepsizes Δx and Δy and the timestep Δt have to satify:

$$\Delta t\,(\varepsilon_{xx}(\Delta x)^{-2}+\varepsilon_{yy}(\Delta y)^{-2})<\tfrac{1}{2} \qquad (26)$$

$$(c_x c_y\Delta t-\varepsilon_{xy})^2-(c_x^2\Delta t-\varepsilon_{xx})(c_y^2\Delta t-\varepsilon_{yy})<1 \qquad (27)$$

Calculations of velocity distributions and bed levels

To test the quality of the flow models calculations have been executed for a flow through a LFM (Laboratory of Fluid Mechanics of the Delft University of Technology) flume (Figure 2). For this bend measurements have been performed with a fixed and flat bed (de Vriend, 1979). Apart from the flow models discussed before, also a flow model has been used in which advective, pressure and bottom friction terms are included (Model IV). In Figure 4 the calculated and measured longitudinal velocity distribution are given for several cross sections. None of the numerical results is quite satisfactory. At the start of the bend the location of maximum longitudinal velocity tends to approach the inner side as the measurements do, but the gradual displacement of this location to the outer side is badly reproduced, as is the slow return to the uniform conditions at the outflow side. It is not sure whether, for an uneven bed, the deviations will be similar, as interaction with depth variations might be more important. The deviations may be ascribed to the neglected influence of the spiral motion on the depth-averaged velocities.

For axisymmetric curved channels the solution method is significantly simplified and analytical methods easily provide insight into the problem. For steady flow conditions the equilibrium bed level is obtained when the sediment transport normal to the flow direction vanishes. From formula (7) expressing the direction of transport it can be derived that the transversal transport vanishes when the transversal component of the gravity force is compensated by the bottom shear force normal to the flow direction. This shear force depends completely on the way in which the spiral motion has been introduced. A number of measurements has been performed for example by Zimmermann (1974), for this type of bends. In Figure 3 calculated bed levels are compared with these experiments (Koch, 1980). There is a slight difference between the different flow models and a good correspondence to the measurements. It suggests that the way in which the effect of the spiral motion on sediment transport has been incorporated can be used.

For each of the three flow models predictions have been made of the bed level distribution in the LFM flume with a movable bed and steady flow conditions. The bed levels and the corresponding velocity distributions are shown in Figure 5. The bed levels do not represent the equilibrium bed levels. With regard to the large differences between the calculated velocity distributions, the differences

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VRIEND, H.J. de: Flow measurements in a curved rectangular channel, Delft University of Technology, Department of Civil Eng., Report 9-79, 1979.

ZIMMERMANN, C.: Sohlausbildung, Reibungsfaktoren und Sedimenttransport in gleichförmig gekrümmten und geraden Gerinnen, University of Karlsruhe, Ph.D. dissertation, June, 1974.

between the bed levels are relatively small. This indicates the influence of the spiral motion on the bed level distribution. Furthermore the case of an uneven bed appears to be more relevant for the evaluation of the qualities of the different flow models than a flat bed. Unfortunately for such a case no measured velocity data are available. As the maximum longitudinal velocity in a cross-section occurs at the inner wall through the whole bend, even for a uneven bed, model I is not suitable for these applications. The differences between the measured and computed bed level are not negligibly small at all locations, as shown in Figure 6. The averaged slope of the bed level is of correct order. However, contrary to the measurements, the calculated bed level shows a rather symmetric configuration. Presumably the asymmetry is introduced by the spiral motion, which is not reproduced correctly by the numerical model. For symplicity reasons the local curvature of the river fitted polar coordinate system has been substituted in formula (10) instead of the local curvature of the stream lines. A more important reason is the neglect of the inertia of the spiral motion, that prevents a direct adjustment to the depth averaged main flow. This effect may explain the shift of the steepest bed gradient in a cross section downstream of the sudden jump of curvature of the inflow part. At present the possibilities to introduce these effects are being investigated. The used bend is of a rather strong curvature. Application of the present method to moderate curved bends may be more succesful and more relevant to natural river conditions.

Conclusions

The used flow models do not allow a correct prediction of the flow field. The neglected effects being the turbulent shear forces and the effect of the spiral motion on the main flow.

The character of the system of equations and consequently its boundary conditions and solution method depends strongly on the used sediment transport formula.

For axisymmetric bends the prediction of the bed configuration is correct and justifies the way in which the spiral flow has been accounted for in the formula for the bed shear stress.

For a strongly curved 180^{o} bend the results are not satisfactory. In these cases the spiral motion is not described correct. Inertia effects and the curvature of streamlines should be included. A more sophisticated flow model may still be required, although it is not certain that this must be a 3-D flow model.

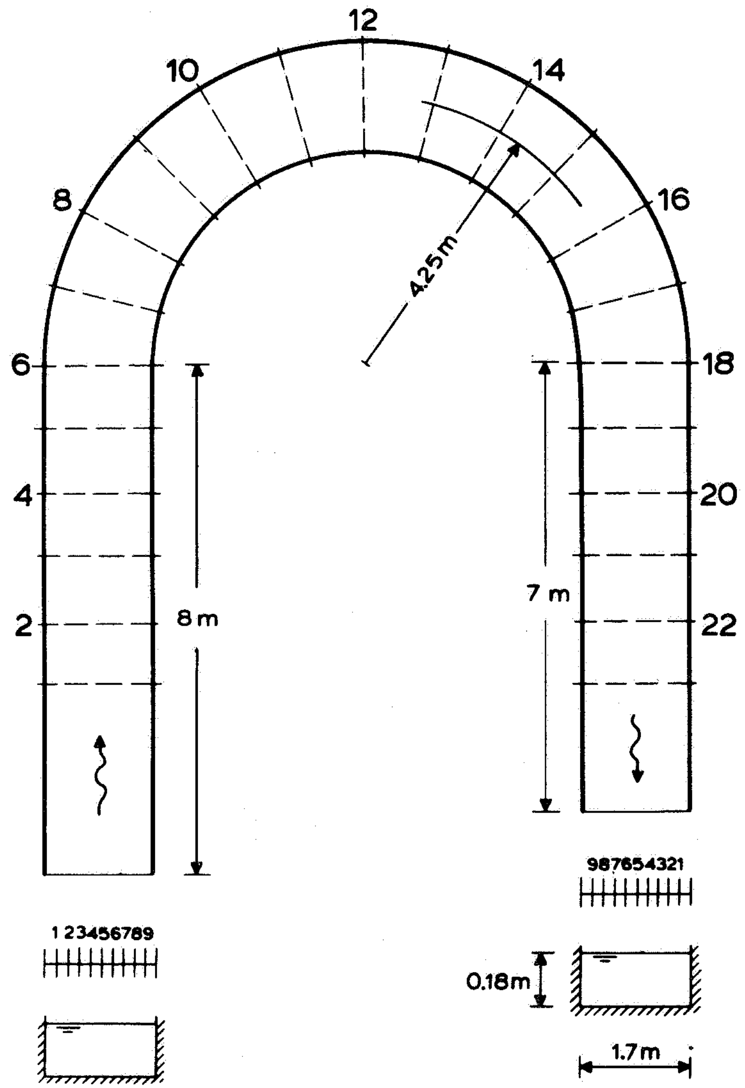

FIG. 2 GEOMETRY OF THE LFM FLUME

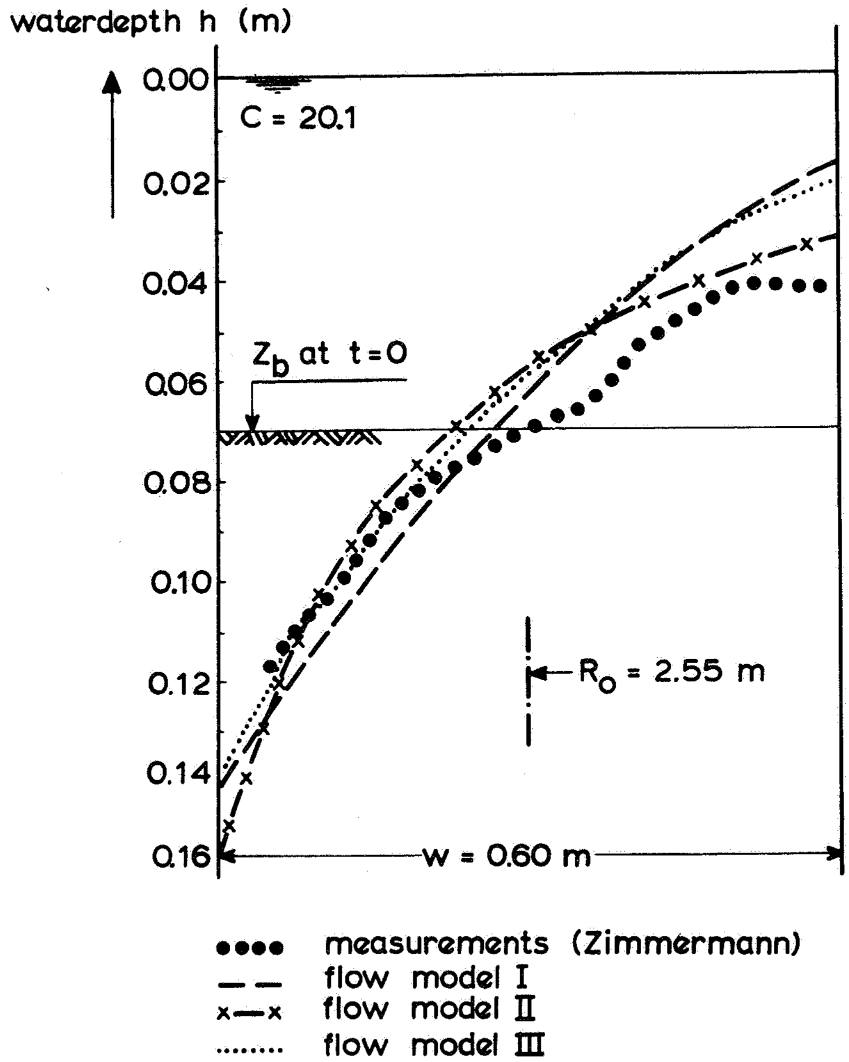

FIG. 3 EQUILIBRIUM TRANSVERSE BED PROFILE

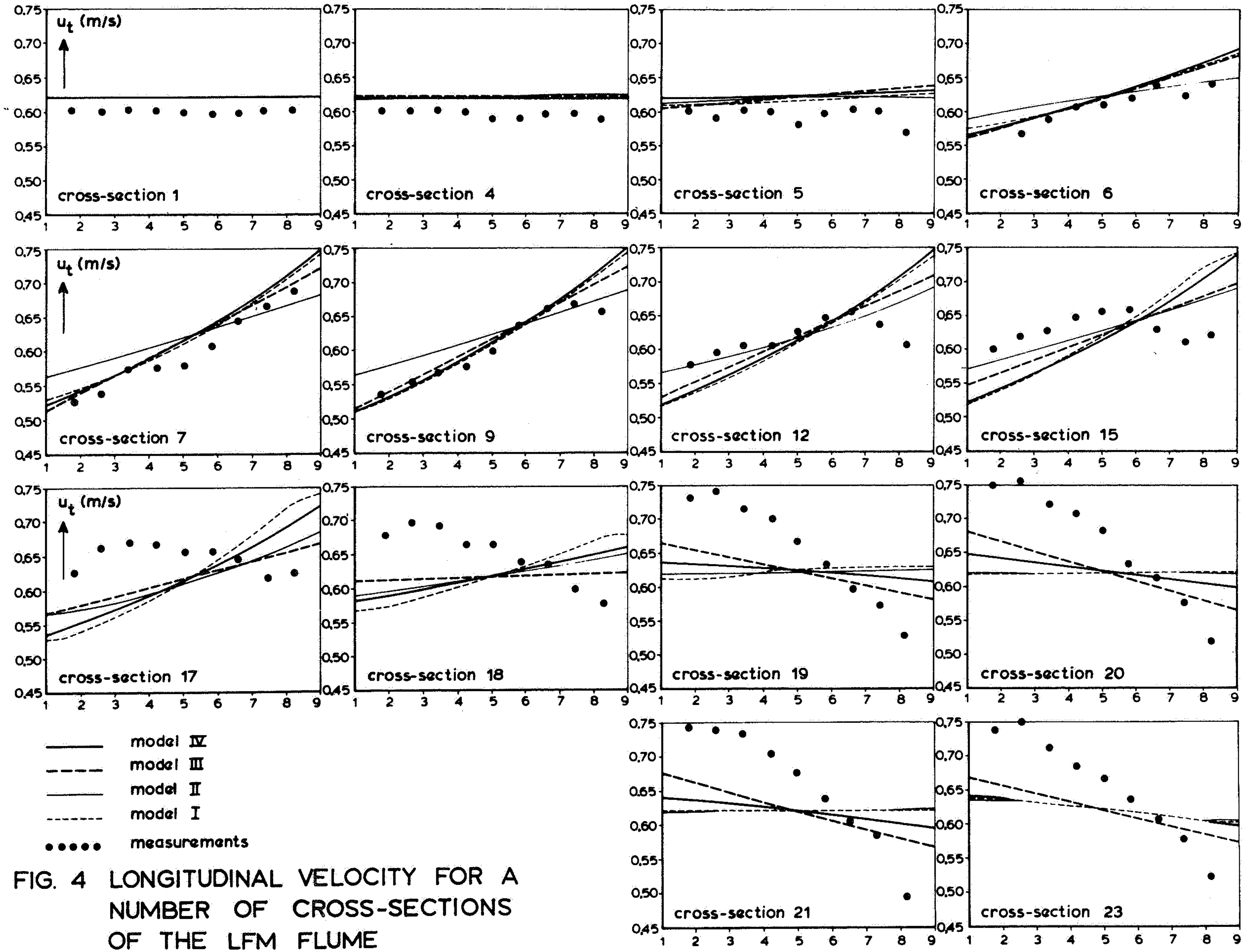

FIG. 4 LONGITUDINAL VELOCITY FOR A NUMBER OF CROSS-SECTIONS OF THE LFM FLUME

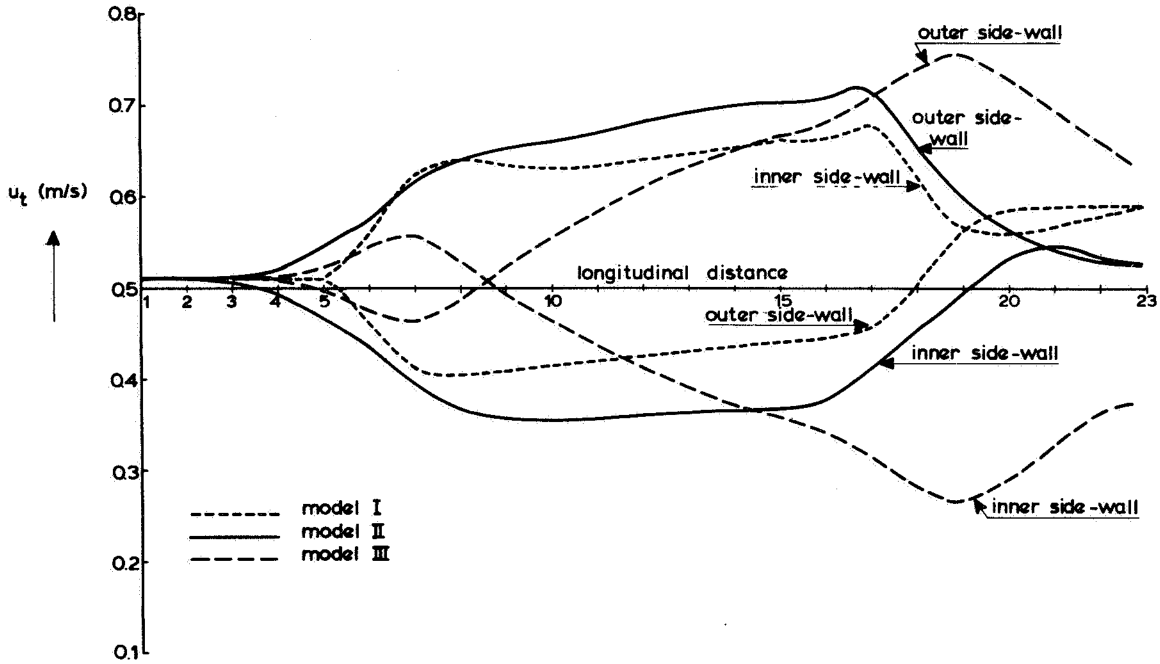

FIG. 5a LONGITUDINAL VELOCITY DISTRIBUTION ALONG THE INNER AND OUTER SIDE-WALL OF THE LFM FLUME

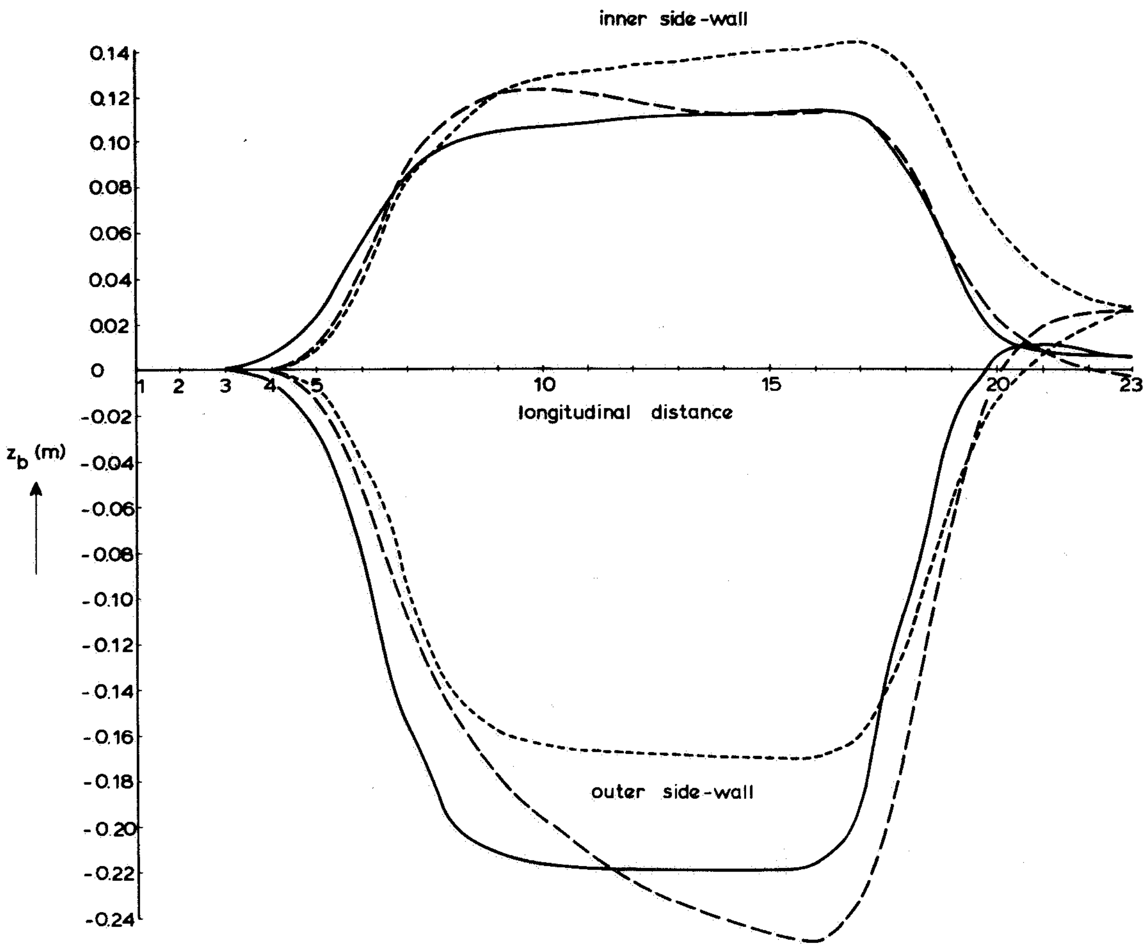

FIG. 5b BED LEVEL DISTRIBUTION ALONG THE INNER AND OUTER SIDE-WALL OF THE LFM FLUME

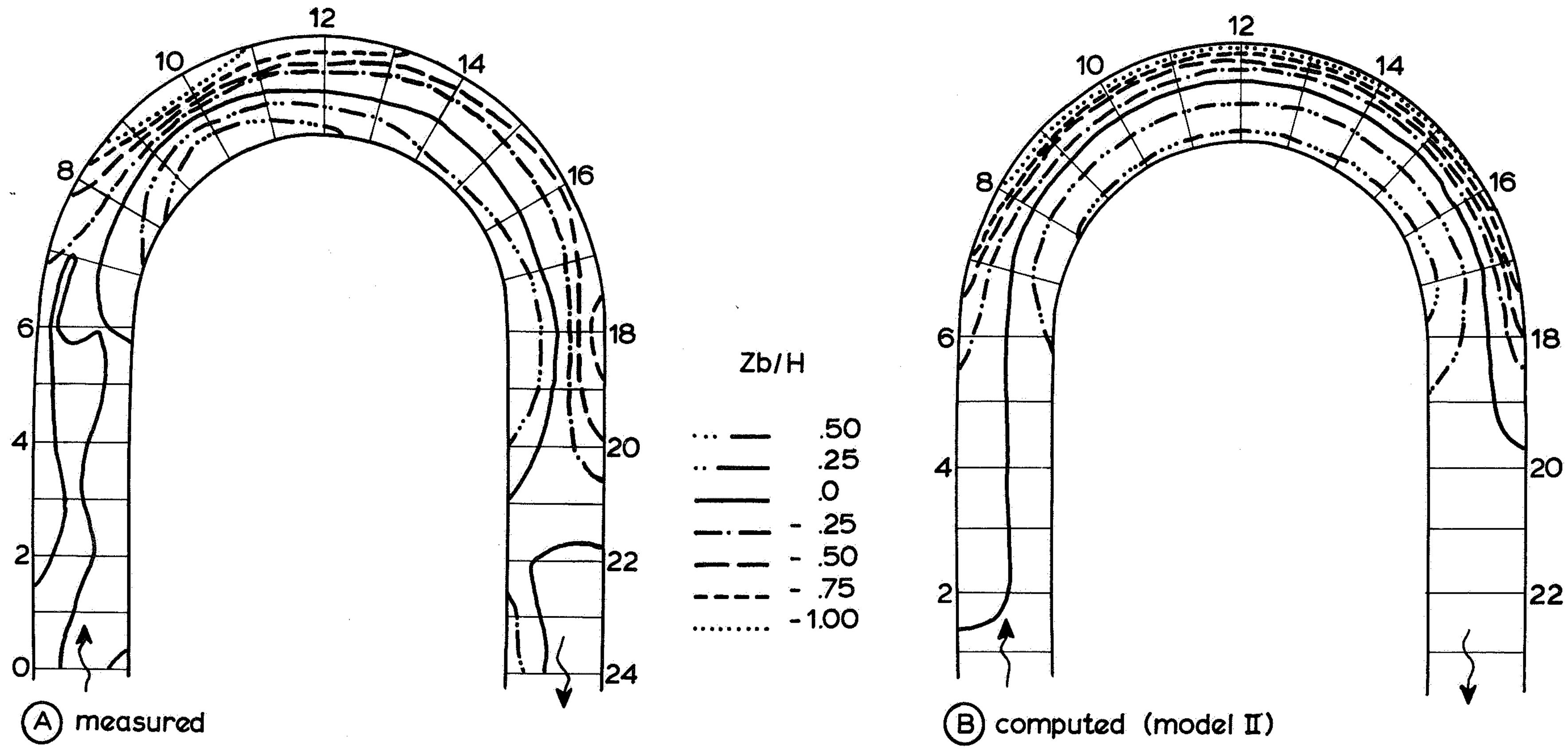

FIG. 6 EQUILIBRIUM BED TOPOGRAPHY (LFM FLUME)