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WAVE KINEMATICS AND FLUID LOADING IN IRREGULAR WAVES

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Abstract

A persisting problem in the offshore world, for engineers in industry and researchers alike, is the hydrodynamic loading on offshore structures in their natural environment. Obviously this problem area is also a major subject within the Netherlands Marine Technological Research programme (MaTS). The problem consists of two parts, i.e. first to obtain an adequate theoretical description of the wave kinematics and second to describe the forces produced by given kinematics. The kinematics problem was investigated first with the emphasis of the investigation being placed on the systematic checking of the validity of the probabilistic, Gaussian random wave model and the establishing of the limits to its accuracy.

Simultaneous measurements of surface elevation and horizontal and vertical velocities were performed in a flume of the Delft Hydraulics Laboratory. Laser-Doppler equipment was used for the measurement of orbital velocities. Nine wave spectra were investigated covering a wide range of sea state severities, including extreme conditions. Results of the measurements in a vertical and a horizontal plane are analyzed in a statistical and spectral manner and compared with theoretical calculations. Furthermore, time series for certain quantities are calculated from a particular measured other quantity. These time predictions are subsequently compared with the measurements. Finally, some results of measurements within the free surface zone and comparisons of measurements with calculations using periodic wave theories are presented.

The results of the investigation support that the linear random wave model can be used quite satisfactory for the calculation of wave kinematics in most engineering applications, and for the full range of sea states covered. Rather obvious limitations of the model concern the kinematics in the free surface zone and the accurate prediction of the tails of the statistical distributions for both the surface elevation and velocities. Enhancements to the linear theory in these areas would be desirable. Another area for further work is the observed existence of low frequency wave groups and their influence.

1. Introduction

A persisting problem in the offshore world is the fluid loading on offshore structures in their natural environment. It has intrigued researchers and engineers in industry for over 30 years and continues to do so. All these efforts illustrate two things, i.e. the very great practical importance of the subject matter and its inherent complexity.

When the Netherlands Marine Technological Research Programme (MaTS; [6]) got underway the subject naturally presented a challenge to the Steering Group for Fluid Mechanics Research. After careful consideration it was decided to address the problem in a systematic and rather fundamental manner. This paper describes the general considerations underlying the approach adopted, the present status of the programme and the results obtained in the first phase which concerns wave kinematics.

The question engineers are faced with is what the fluid loads are in the natural environment in which the structure exists, that is in random sea states. Simplifications to this overall picture are only useful if it can be demonstrated that such an idealisation is still an adequate representation of reality for the problem under consideration, or if the idealisation is a building block in a more comprehensive model [11]. The building block approach is not a simple one, though, as superposition is usually not considered to be permissible due to the apparent, or at least suspected, non-linearities in both the environmental and the force descriptions.

The overall problem consists of two parts, i.e. first an adequate theoretical description of the wave kinematics in a natural sea (unaffected by the presence of the structure) and second a mathematical formulation of the forces produced by given kinematics. It was decided that the problem of wave kinematics should be resolved first and the investigation undertaken to do this forms the main subject of this paper. The second problem constitutes undoubtedly an even greater challenge, especially when considering jacket type structures. This part is to be addressed during phase 2 of the investigations, which is about to start (see also Section 3).

2. Wave kinematics in a natural sea

It is essential for the entire discussion which follows that it is fully appreciated that the purpose of the investigation is to determine the best possible description of the flow field in the natural offshore environment. If randomness is a significant feature of the phenomenon, then this aspect should be duly incorporated. Similarly, there are clear indications that multi-directionality or wave spreading play an appreciable role offshore [4]. On the other hand, if non-linear phenomena are important factors these should be properly formulated and included. If the present state of the art does not permit to include all these aspects equally well then preference should be given to what are found to be the most important ones.

For practical applications the problem may be formulated in general terms as follows: "Develop a description of the water particle velocities and accelerations which exist in a particular sea state as a function of time, and over a three-dimensional region of the size of the (largest) dimensions of an offshore structure. This should be accomplished with an accuracy which is sufficient for the subsequent determination of the fluid loads on the structure within a margin which is acceptable for engineering applications."

There are currently two theoretical models available. First the deterministic, periodic wave model, which may incorporate higher order terms but cannot account for randomness nor directionality. Second the Gaussian random wave model, which has no difficulty with the inclusion of the two last mentioned aspects, but which is essentially based on linear superposition and consequently lacks in detail represented by non-linear formulations such as required for the area around the still waterline between the trough and the crest of the wave, and as may be expected to be significant for high waves in general.

To study the adequacy of these two theoretical models an extensive and carefully controlled experimental investigation at laboratory scale was considered desirable and necessary. It was anticipated that the Gaussian random model was more powerful and had a greater potential to satisfy the problem formulation given above. Therefore, the emphasis of the investigation was placed on the systematic checking of the validity of this linear model and the establishing of the limits to its accuracy. This was to include the important area around the mean still water level where linear theory is by its very nature inadequate. Furthermore, the experimental verification should include a large range of sea state severities, from mild to extreme. It was duly recognized, however, that in the absence of adequate measured data the anticipation of the most appropriate theoretical model depends on the judgement of the assessor and thus may vary from person to person. Consequently, the applicability or otherwise of the deterministic, periodic wave model for the same range of conditions should be assessed with equal precision.

It appears that the essence of the different judgements is related to the relative importance of errors in either the free surface boundary condition (which would favour non-linear periodic wave theories) or in the connectivity conditions between individual waves in the vertical cross sectional planes. Therefore, the results of the investigation implicitly provide an answer to this question.

An experimental investigation at laboratory scale is confined to uni-directional waves so that the aspect of wave spreading cannot be included. However, the extension from two to three dimensions is not believed to introduce any essentially new aspect in the matter, other than that periodic wave theories cannot cope with this additional aspect.

3. Wave forces in a natural sea for given wave kinematics

While conducting the kinematics study, phase 2 of the overall programme was not forgotten. Shell Internationale Petroleum Maatschappij B.V. (SIPM) and its consultants had during several years worked on the development of non-linear random data analysis techniques which would be capable of separating and quantifying linear and non-linear components of fluid loads in the random environment [11]. Examples of such components are inertial and drag forces according to the traditional Morison equation, but non-linearities should not necessarily be thought of as having this form or being limited to these traditional formulations. The work had progressed to the stage where application of the theoretical developments to controlled measurements was in order. These developments were considered to be fully in line with the outcome of the kinematics study, which was expected and indeed appeared to favour the Gaussian random model. Therefore the MaTS organisation and Shell (SIPM) decided to jointly undertake phase 2 of the programme. A detailed proposal for the measurement of wave forces on an element of a vertical cylinder in irregular waves, again at laboratory scale, was submitted to the Dutch Ministry of Economic Affairs for approval as a project in the MaTS-programme. This approval has now been given, and the project is planned to start in the second half of 1981. It should be noted that this project is aimed at establishing the physical force mechanisms and their formulations, and should not necessarily be expected to provide immediate quantitatively applicable results for the real offshore case.

4. Manner in which the kinematics study has been conducted

Nine wave conditions have been selected (see Table 1) varying from deep water to transitional conditions. The sea states are grouped in three groups having different periods of the spectral peak. Within each group three significant wave heights have been chosen, up to the maximum wave steepness which could physically be realized without waves breaking.

As stated before, the main objective of the present investigation is to verify the validity of the linear random wave model via controlled laboratory experiments in uni-directional, irregular waves. During the experiments simultaneous measurements were performed of the surface elevation and of horizontal and vertical velocities using Laser-Doppler instrumentation. Measurement locations included 6 positions along a vertical axis over the water column, and 3 positions along a horizontal axis at an elevation of 0.75 of the water depth; see further Section 6 of the paper.

The verification is performed in three different stages, which are increasingly demanding on the accuracy of the theoretical calculations which are compared with the measured data. These are:

a) Comparison of the statistical distributions of measurements and calculations for instantaneous values and peak values of all parameters. This statistical analysis involves one signal at a time and does not relate any one parameter with another one.

b) Comparison of gain and phase of the frequency response functions obtained by spectral analysis of pairs of measurements with the theoretical predictions. This part of the analysis relates two chosen parameters to one another in the frequency domain.

c) Comparison of a measured parameter as a function of time with a time domain prediction of the same parameter, calculated from another measured variable by transformation using strictly linear techniques. Although, for a linear system, the time domain representations of the parameters are uniquely related to the frequency domain representations of item b) above, this test is a very demanding test in which inaccuracies or effects of non-linearities will show up rather strongly. This time domain comparison was also considered to provide the ultimate and most convincing demonstration of the capability (or lack of it) of the linear random wave model.

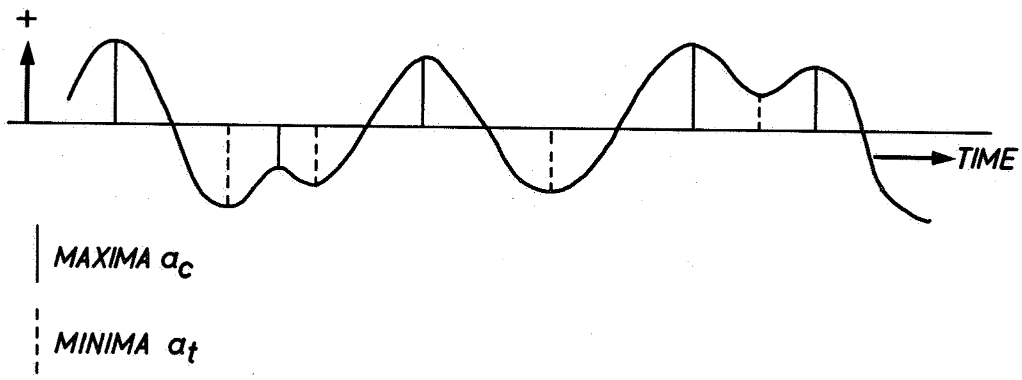
5. Theoretical background

5.1. Statistical distributions

As already mentioned before the statistical distributions of instantaneous values and peak values will be compared with theoretical distributions. The mean record level is used as reference level. Parameters that have to be used in the calculation of the theoretical distributions are derived from the measured time series.

The statistical distribution of instantaneous values is of course compared with the Gaussian or normal distribution. This distribution is completely defined by the standard deviation σ, which is equal to $\sqrt{m_o}$.

Maxima a_c and minima a_t are defined according to the definition sketch below. It should be noted that maxima can be negative and that minima can have a positive



value. The theoretical distribution for these peak values was derived by Rice [8] and further elaborated by Cartwright and Longuet-Higgins [2]. This distribution is defined by two parameters - the standard deviation σ and the spectral width parameter ε which can either be calculated from the spectral moments of the energy density spectrum (ε_m) or from the mean periods of maxima and zero-crossings (ε_T). In theory these two spectral parameters should be the same but in practice there may be great differences due to the fact that ε_m is very sensitive to variations in the calculation procedures, high-frequency cut-off, etc. [9]. For the case that ε approaches zero the Rice distribution approaches the well-known Rayleigh distribution; for the limiting case of $\varepsilon = 1$ it corresponds to the normal distribution.

5.2. Frequency domain analysis

Gain and phase functions of two measured time series are calculated by means of cross spectral analysis (see [1]) and compared with linear theory. According to this theory the gain of the complex frequency response funtions between the surface elevation η and the horizontal velocity u and between η and the vertical velocity w in a vertical plane is given by, respectively,

$$|H_{\eta u}(f)| = \frac{2\pi f \cosh(2\pi a/L)}{\sinh(2\pi d/L)} \tag{5.1}$$

and

$$|H_{\eta w}(f)| = \frac{2\pi f \sinh(2\pi a/L)}{\sinh(2\pi d/L)} \tag{5.2}$$

The phase relations between η and u and between η and w are respectively 0^o and -90^o.

The frequency response functions obtained by spectral analysis of pairs of time series at measurement points which are separated over a certain horizontal distance are also analyzed. In uni-directional seas the gain function is then theoretically equal to unity, while the phase function is linearly dependent on the ratio between the horizontal separation and the wave length.

5.3. Time domain calculations

The relationship between two quantities is considered to be linear. Surface elevation (input) and velocities (output) in a vertical plane are then related by

$$u(t) = \int_o^\infty h_{\eta u}(\tau) \left[\eta(t-\tau) + \eta(t+\tau)\right] d\tau \tag{5.3}$$

and

$$w(t) = \int_o^\infty h_{\eta w}(\tau) \left[\eta(t-\tau) - \eta(t+\tau)\right] d\tau \tag{5.4}$$

where $h_{\eta u}(\tau)$ and $h_{\eta w}(\tau)$ are impulse response functions. These impulse response functions are the inverse Fourier transforms of the frequency response functions (see Section 5.2). Thus, for a given surface elevation a time serie of u or w at any elevation can be calculated through (5.3) and (5.4).

The same technique is used to calculate from one point to another point, separated by a certain horizontal distance. The formula for these calculations is given for surface elevations; entirely similar equations are valid for the velocities at two points:

$$\eta_2(t) = \int_{-\infty}^{\infty} h_{\eta\eta}(\tau)\eta_1(t-\tau)d\tau \qquad (5.5)$$

where $h_{\eta\eta}(\tau)$ is the impulse response function, $\eta_1(t)$ is the input surface elevation and $\eta_2(t)$ is the output surface elevation.

Details of these time domain calculations are given in [1,10]. Two comments should be made:

a) The upper frequency for the calculations involving the inverse Fourier transforms of the frequency response functions was fixed at 2.5 Hz. Above this frequency the input spectra did not contain any energy.

b) The upper limit of the integrals in (5.3) and (5.4) is given as $+\infty$; the lower and upper limits of the integral in (5.5) are $-\infty$ and $+\infty$. This is correct in theory but for numerical computations the impulse response functions have been put equal to zero above a certain time t_o in (5.3) and (5.4) and below $-t_o$ and above $+t_o$ in (5.5). Time t_o is chosen in such a way that $h(t_o)$ is less than 1% of the maximum value at any time t between o and t_o in (5.3) and (5.4) and between $-t_o$ and $+t_o$ in (5.5).

5.4. Periodic wave theories

Three deterministic, periodic wave theories are selected for comparison with individual waves chosen out of the measured time series. These theories are the linear theory, the 3rd order Stokes theory according to Laitone [7] and the 5th order Stokes theory according to Dé [3]. Details of the theories can be found in the references and will not be repeated here.

6. General description of the experiments

6.1. Model set-up

All measurements were performed in a 50 m long and 1 m wide wave flume of the Delft Hydraulics Laboratory at De Voorst ("Scheldegoot"). This flume is equipped with a wave generator at one end and a wave-absorbing beach (slope 1 to 10) at the other end (Figure 1). The wave generator is able to generate both regular and irregular waves. In order to reproduce the orbital velocity at the flap as closely as possible the ratio of rotation and translation of the wave generator was adjusted for every test (cradle-type wave generator).

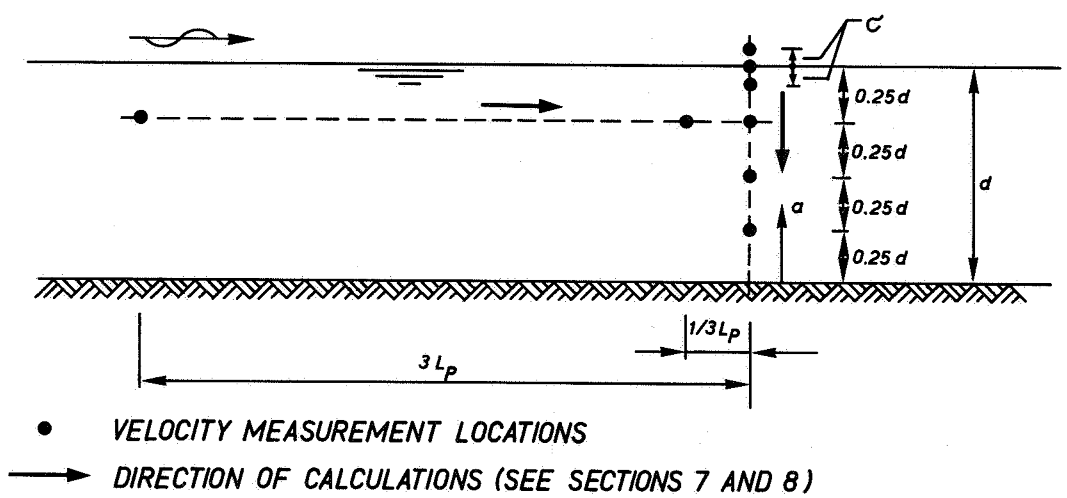
The wave generator is driven by an electronically controlled actuator. Input for the electronic control unit is an analog signal. In this way any desired programme of waves can be generated. For this investigation pseudo white-noise (frequencies between 0 and 3 Hz) was filtered in such a way that the required energy density spectrum was obtained. Time series representing the desired wave spectrum were recorded on magnetic tape and used as input for the wave generator. In this way it was possible to repeat a test several times without any significant difference between the time histories of the tests.

During all tests the waterdepth was fixed at 0.85 m and no wind was applied. Orbital velocities were measured by means of two Laser-Doppler velocity meters. Each of the velocity meters could measure the horizontal and vertical orbital velocity at the same time and place. The set-up of the velocity meters is shown in Figure 2. The two Laser-Doppler velocity meters that were used are of the forward scatter type with reference beam. The frequency of one of the Laser beams is shifted due to the movements of water particles. This shift is directly related to the orbital velocity at the point of measurement. More details are described in [5]. The surface elevation was measured by means of a resistance type wave height meter. During all tests simultaneous recordings of the surface elevation and the orbital velocities were made in the cross-sections of the flume where the velocities were to be measured.

6.2. Programme of tests

Wave spectra varied from transitional to deep water conditions and up to maximum wave steepness. Breaking wave conditions were not included. Significant parameters of the nine wave spectra that were investigated are given in Table 1, while the energy density spectra of the surface elevations are presented in Figure 3. The water depth - wave length ratio d/gT_p^2 varies from 0.02 to 0.10, while the wave steepness parameter H_s/gT_p^2 is chosen between 0.0014 and 0.0077.

Orbital velocities in a vertical plane were measured at 6 locations (see definition sketch). Three of these locations are at a/d = 0.25, a/d = 0.50, a/d = 0.75,



where a is the distance above sea bed and d is the water depth. Furthermore, around the still water level velocities were measured at a distance of σ below and above this level and just at the still water level. The standard deviations were calculated from the measured time series of the surface elevations.

Two horizontal separations were chosen and related to the wave length L_p which was calculated for the spectral peak period T_p (see Table 1): 1/3 L_p and 3 L_p. These separations are 0.447 m and 4.05 m for wave spectra 31-33, 0.833 m and 7.50 m for wave spectra 41-43 and 1.483 m and 13.35 m for wave spectra 51-53. The velocities were measured at one elevation only, i.e. at a/d = 0.75 (see definition sketch).

6.3. Experimental procedure and data processing

For each wave spectrum five tests were performed with the same steering signal for the wave generator, but with different positions of the velocity meters and wave height meter(s). During each test the orbital velocities were measured simultaneously in two points, either both situated in a vertical cross-sectional plane or separated over a certain horizontal distance. The results of the tests in the vertical plane are described in Section 7, while results of measurements with a horizontal separation are given in Section 8.

During each test the signals were simultaneously recorded on magnetic tape. The five tests of each wave spectrum were synchronized by a starting point on the tape locating the same position at each of the recorded surface elevations. In this way it was possible to analyze the same part of the time series for each of the tests. The duration of each test was 35 minutes. After digitisation with a sample frequency of 25 Hz each time serie consisted of 53248 data points. Before digitisation all signals were filtered analog at 10 Hz Low Pass in order to avoid problems with the Nyquist-frequency at 12.5 Hz. After digitisation a digital Bandpass filter was applied on all time series before further analysis took place. The lower frequency of this filter was chosen at approximately 0.05 Hz because of the fact that the spectral calculations did not show any information on longer periods in the signals. Thus the effect of long periodic waves in the flume was eliminated and also the effect of a possible electronic drift of the measurement equipment. The upper frequency of the digital filtering was fixed at 2.5 Hz. This was allowable because there was no noticeable energy present above this frequency. Filtering was mainly done because of the rather strong effect of high-frequency noise on the higher spectral moments, and thus on the spectral parameter ε_m. The basic principle of the filtering applied was, therefore, to remove all Fourier components below 0.05 Hz and above 2.5 Hz.

7. Results of the measurements in a vertical plane

7.1. Statistical analysis

7.1.1. Instantaneous values

An example of the statistical distributions of the measured instantaneous surface elevation and orbital velocities is given in Figure 4 for wave spectrum 43, together with the theoretical distributions. These figures are typical for all measurements and calculations. Table 2 summarizes results for all wave spectra. The results in Table 2 are presented as the relative error between measurement and theory in the tail of the distribution, and are expressed in % of the theoretical value at 1% exceedance. A positive percentage means that the measured values are greater in magnitude than theory predicts. Positive surface elevations are measured upwards from the still water level. A positive velocity means for the horizontal velocity a velocity in the direction of wave propagation and for the vertical velocity a velocity in upwards direction.

Figure 4 demonstrates that surface elevation and orbital velocities on the whole conform to the theoretical Gaussian distribution quite well. Generally speaking the deviations are greatest for the surface elevations, but they are confined to 1% of the troughs and 5% of the crests. The horizontal velocities differ more under the 5% largest troughs than under the 5% largest crests. The vertical velocity shows very good agreement with theory over the entire range of velocities. A measure of the degree of difference in the tails of the distributions will be discussed below in somewhat greater detail.

From Table 2 it is clear that the measured surface elevation differs most from theory for the steepest waves (spectra 33, 43 and 53) at a probability level of 1%. Deviations from theory start for the positive surface elevations at 5% exceedance. Especially for the somewhat longer period wave spectra theory predicts greater negative surface elevations than follows from the measurements, which is probably due to the influence of the bottom of the flume. As Table 2 indicates positive horizontal velocities are smaller in magnitude than will be calculated with the linear theory. The relative error increases with wave steepness. This is also the case for the negative horizontal velocities, but here the measurements give much higher velocities than theory predicts. The measured horizontal velocities differ most from theory, percentagewise, for the lowest elevation ($a/d = 0.25$). For almost every wave spectrum and at all elevations the measured vertical velocities exceed the theoretical values at a probability level of 1%. The magnitude of the relative error goes up to about 5%, except for the deep water spectra 31-33 for the lowest two elevations, where the differences are somewhat greater. It should be noted that in these cases the vertical velocities are rather small.

7.1.2. Maxima and minima

The definitions of maxima (crests) and minima (troughs) are given in Section 5, while definitions for positive and negative directions and the relative error are described in the previous Section. Some results of the measurements are presented in Figure 5 (again for wave spectrum 43). Minima are in this Figure presented with reversed sign. Table 3 summarizes for all wave spectra the relative errors at a probability level of 1%. Measured values are compared with the theoretical Rice distribution. The spectral parameter ε is calculated from the spectral moments between 0 and 2.5 Hz.

The overall results are similar to those for the distributions of instantaneous values, but with in most cases a somewhat greater magnitude of the relative errors in Table 3. Generally speaking, deviations from theory are greatest for surface elevations; this is especially so for the wave crests, the distribution of which is consistently higher than the distribution of wave troughs. The deviations also increase with the steepness of the sea state maintaining the same peak period.

For most wave spectra there is a specific percentage at which the distributions of the horizontal crest and trough velocities cross each other (see Figure 5). This percentage shifts from 35% for the deep water wave spectra to 25% for the transitional water wave spectra. This means that horizontal velocities under larger wave troughs are greater than velocities under larger wave crests. For the upper tails of the distributions (see Table 3) the crest velocities are generally smaller than theory predicts, whereas the trough velocities exceed the theoretical magnitude. However, there is no strictly consistent trend of increasing deviations from theory with increasing steepness of the sea states (maintaining the same peak period) as there is for surface elevations.

The agreement between experiment and theory for the vertical velocities is on the whole very good. The detailed comparison at 1% exceedance level in Table 3 also indicates that the differences are usually appreciable smaller than for the horizontal velocities. The maxima of the vertical velocity exceed the minima at this probability level.

The Rice distribution predicts the overall nature of the measured distributions certainly better than the Rayleigh distribution for all parameters. However, the difference between the two for the lower exceedance levels becomes smaller and smaller, as is well-known. Therefore, there is no significant difference between the two for a prediction of the upper tail of the measured distributions.

7.2. Spectral analysis

7.2.1. Gain functions

For comparison of gain functions derived from the measurements and linear theory a relative error ξ was defined by

$$\xi = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{|\hat{H}_j(f)| - |H_j(f)|}{|H_j(f)|} \right| \qquad (7.1)$$

where $|\hat{H}(f)|$ = gain function from the measurements

$|H(f)|$ = theoretical gain function (linear wave theory)

N = number of frequencies for which gain function estimates are included in the comparison.

The theoretical gain functions were calculated by means of (5.1) and (5.2). The gain functions from the measurements were obtained from the cross spectral analysis of the data. The gain function between, for instance, the surface elevation (input) and the horizontal velocity at a certain depth (output) is given by

$$|\hat{H}_{\eta u}(f)| = \frac{|S_{\eta u}(f)|}{S_{\eta\eta}(f)} \quad . \tag{7.2}$$

where $|S_{\eta u}(f)|$ is the amplitude of the cross-spectrum of input and output, and $S_{\eta\eta}(f)$ is the energy density of the input. The spectra are computed using a Fast Fourier Transform. The squared coherence function $\gamma^2_{\eta u}(f)$ can also be calculated from the spectra of input and output:

$$\gamma^2_{\eta u}(f) = \frac{|S_{\eta u}(f)|^2}{S_{\eta\eta}(f).S_{uu}(f)} \quad . \tag{7.3}$$

Details are given in [1].

The relative errors ξ have been calculated only for those frequencies f for which $\gamma^2(f) \geqq 0.90$. Over this frequency range the system behaves rather like a linear system.

The results of the calculation of these relative errors are presented in Table 4, while Figure 6 shows the gain functions themselves for wave spectra 33, 43 and 53. Measured values are indicated only for the frequency range for which $\gamma^2 \geqq 0.90$. This range virtually coincides with the frequency range over which input and output have measurable energy levels and thus provide relationships which can clearly be distinguished from the noise levels involved in any measurement and analysis. As Figure 6 illustrates, the general agreement between measurement and calculations is very good. The largest deviations are found for the deep water spectra 31-33. There is a tendency for the gain functions $\eta \rightarrow u$ and $\eta \rightarrow w$ to be somewhat greater in magnitude than theory indicates for higher frequencies within the range for which $\gamma^2 \geqq 0.90$.

7.2.2. Phase functions

Table 5 summarizes the average values of the differences between the phase relationships derived from the measurements and linear theory for the frequency range for which $\gamma^2 \geqq 0.90$. The phase between the surface elevation and the horizontal velocity in a vertical plane does not differ much from zero degrees; the variation over the frequency range is within a few degrees. This is different for the phase relationship between the surface elevation and the vertical velocity, where the maximum (average) deviation from theory is 21 degrees (wave spectrum 32, a/d = 0.25). It should again be noted that especially wave spectra 31 and 32 produce only rather small velocities, horizontally and vertically, at the elevation a/d = 0.25, so that even large errors correspond with only small differences in the velocities. In the measurements there is a significant tendency for the phase between η and w to increase with increasing frequencies (within the range for which $\gamma^2 \geqq 0.90$).

7.3. Calculated time series

The main principle of the calculation method is given in Section 5. Calculations are again carried out for wave spectrum 43. Both the horizontal and the vertical velocity are calculated at three elevations below the wave trough (a/d = 0.25, 0.50 and 0.75) and at the still water level (a/d = 1.00). The measured surface elevation serves as input for the calculations. Time series with a duration of 20 minutes were calculated. The results of the calculations are compared with the measurements in two ways. Table 8 presents some information about the differences between the statistical distributions of instantaneous values and maxima/minima that were derived from the measured and calculated time series. In Figure 7 typical parts of the time series are presented in order to make the comparison between measurements and calculations more visible.

From the comparison of the statistical distributions derived from measurements and calculations the conclusions can be drawn that in the statistical sense there is a rather good agreement between measurements and calculations. This agreement is even better then between measurement and purely theoretical distributions. This is indicated by comparing the relative errors presented in Table 8 with similar results presented in Tables 2 and 3. Especially the calculated horizontal trough velocities fit the measurements better.

The results presented in Figure 7 illustrate the rather good agreement between measurements and calculations, especially as far as the records below the wave trough are concerned. This is of course not at all surprising in view of the previous discussion in this Section.

The results of the measurements and calculations at a/d = 1.00 (still water level) show that the maximum horizontal crest velocities from the calculations are higher than the measured velocities. For those parts of the record of the vertical velocity for which measurements and calculations can be compared the results indicate a rather good agreement. It should be noted that due to transients in the Laser-Doppler signals the actual period over which reliable measurements could be obtained is of the order of 0.10 to 0.15 s shorter than the period for which $\eta \geqq 0$.

7.4. Individual large waves

One individual wave has been selected out of the time series representation of wave spectrum 43. The trough-crest height of this wave is H_z = 0.24 m with a zero-crossing period T_z = 1.26 s. This wave is the part of the time series given in Figure 7 which is marked by the square. Calculations are performed for three wave theories: linear theory, 3rd order Stokes theory and 5th order Stokes theory (see also Section 5.4). The results of these calculations coincide for the higher order theories for the specific H_z-T_z combination mentioned above. It should be noted that calculations have so far only been made for one wave and therefore hardly any general conclusion can be drawn from this result.

The results of the calculations are presented in Figure 9. These results show that the amplitudes of the surface elevation are calculated well by the higher order theories. This is also true for the amplitudes of the vertical velocities, but not for the horizontal velocities, where the measurements show greater trough velocities and smaller crest velocities than theory predicts. This is in close agreement with the results presented in Figure 7. Both the surface elevation and the velocities show rather large phase differences between measurement and theory at zero-crossing points. These differences seem to be much larger than may be observed in Figure 7, where linear random theory is used for the calculations.

8. Results of the measurements over a horizontal separation

8.1. Gain and phase functions

The relative errors between the gain functions derived from the measurements over a horizontal separation of 1/3 L_p and the theoretical gain functions are given for all wave spectra in Table 6 and are calculated using (7.1). These errors are rather low in comparison with the relative errors presented in Table 4 (gain functions in a vertical plane). Within the frequency range for which $\gamma^2 \geqq 0.90$ the gain functions for the surface elevations and the horizontal velocities are somewhat less than one, whereas the gain function for the vertical velocities varies around one or is somewhat higher. Table 7 presents the results of comparison of the phase relationships for measurements and theory for a horizontal separation of 1/3 L_p in the same manner as done for the measurements in a vertical plane (Section 7.2.2.). These results indicate that the differences between measurements and theory are only a few degrees for frequencies for which $\gamma^2 \geqq 0.90$.

Measurements have also been performed for a horizontal separation of 3 L_p. The results of cross spectral analysis of these measurements show that for none of the wave spectra that were investigated the squared coherence function $\gamma^2(f)$ between two horizontally separated quantities is higher than about 0.5. However, the results of the analysis are influenced by the equivalent time lag between the records. As a result of the dispersion of wave components this time lag varies with frequency, which constitutes a major problem for any analysis involving records taken at different horizontal locations, especially for greater distances between the measurement points. In the case of a separation of 3 L_p the analyzed time series are too short to get a reliable answer, and no conclusions can be drawn with respect to the applicability of the linear model over such distances. The results of these measurements will therefore not be presented in this paper.

8.2. Calculated time series

Calculations were made for the relationship $\eta \rightarrow \eta$ and for the relationships $u \rightarrow u$ and $w \rightarrow w$ at an elevation of $a/d = 0.75$ for a horizontal separation of 1/3 L_p in the direction of wave propagation (see also the definition sketch in Section 6.2). Results are presented in Table 8 and Figure 8.

As far as the statistical distributions are concerned the results are again rather good (see Table 8). The time series presented in Figure 8 also show that the agreement between measurement and theory is quite good. It appears that velocities can be calculated even better than the surface elevation.

9. Discussion

As outlined before, the main objective of this investigation is to verify the validity of the linear random wave model and to establish what the possible limits to its adequacy are for practical engineering applications, notably with regard to the calculations of fluid loads. Therefore the comparisons between periodic wave theories and measured individual waves will not be pursued any further in this paper. Furthermore it should be noted that the results presented here only concern a typical sample of all data gathered. Further analysis is still being performed and a more complete and comprehensive account of the study will be given in a report documenting the project within the framework of the MaTS-programme. It may be expected that this final report will be made publicly available as well. Finally it appears worthwhile to emphasize that the study was conducted for sea states covering the full range of mild to very severe conditions. The examples in this paper concern spectrum 43 which, when scaled up, may be thought of as corresponding with a sea state having a return period in excess of one year for the northern North Sea. Similarly, spectrum 53 corresponds with a sea state higher than the 100 year storm in the northern North Sea. This should be borne in mind in assessing the differences between measurements and linear theory and the adequacy of the linear random model.

Most of the details of the comparison between measurements and linear random theory have already been discussed in the previous two Sections. In this Section some main lines will be drawn.

The overall agreement between the statistical distributions derived from the measurements and the theoretical distributions is quite good. The surface elevations show a crest/trough distortion that can also be found in the calculations with deterministic, higher order wave theories (see Figure 9). This distortion is increasing with increasing wave steepness (maintaining the same spectral peak period). Gauss and Rice distributions predict lower crests and deeper troughs in the tail of the distribution than the measurements show.

The statistical distributions of horizontal velocities show a completely opposite behaviour to that of the surface elevations. The larger trough velocities are much greater than theory predicts, while the larger crest velocities are smaller than follows from theory. Such a distortion will not be found with periodic wave theories. Calculations have shown that crest/trough distortions for the horizontal velocities disappear when low frequency energy is removed from the velocity signals by filtering. This energy is concentrated between 0 Hz and the frequency at which the steeply rising flank of the energy density spectrum starts. The statistical distributions of separate crest and trough velocities then virtually coincide and compare quite favourably with the theoretical Rice distribution. It seems that this low frequency energy is a second order phenomenon associated with the formation of wave groups in the velocity signal, which indeed reduces the crest velocities and increases the trough velocities. In an absolute sense this phenomenon is very similar for all elevations which results in greater relative distortions for the lower elevations. The relative errors generally also increase in magnitude for the steeper (and thus higher) sea states, maintaining the same spectral peak period, but spectra 43 and 53 appear to be an exception to this general rule. See Table 3 for quantitative illustrations of the points discussed. Some further work connected with the above observations is still ongoing, the results of which will be incorporated in the final report.

The agreement between measurements and theory in the statistical sense is rather good for the vertical velocities. Measured velocities are mostly larger than calculated velocities in the tail of the distributions as indicated by Table 3. The distributions of vertical crest and trough velocities are mutually very similar and correspond quite well with the theoretical Rice distribution.

Comparison of measurements and theoretical calculations in the frequency domain (gain and phase functions) show good agreement for relationships between quantities in a vertical plane (velocities below the deepest wave trough) and also between quantities which are horizontally separated over a short distance. Measurements match linear theory best for the highest elevation (a/d = 0.75) and for the wave spectra with longer wave periods. This agrees with the results found by Vis [10].

The results of the time domain calculations are very promising. As far as the velocities in a vertical plane below the deepest wave trough are concerned, it is possible to calculate time series which represent the measured records quite well using the linear random model. The statistical representations of the calculated time series are closer to the statistical distributions of the measured quantities than the purely theoretical distributions, especially for the horizontal trough velocities. This reflects that (part of) the distortion is accounted for by linear operations on the measured water surface elevation. This surface elevation incorporates the effects of second order, wave group phenomena which have a great influence on the horizontal velocities, as discussed earlier in this Section.

Time domain calculations over a short horizontal distance do also give good results (Figure 8). It seems that the dispersion of wave components is rather well described by linear theory over such a short distance.

The calculation of velocities at the still water level show that there is a rather great difference between measurements and calculations in the sense that linear theory overpredicts the measurements for this specific wave spectrum. More data are available but have not yet been analyzed, so that the results presented here can only give an indication of what might be expected. In any event it illustrates the importance of obtaining more knowledge about wave kinematics in the free surface zone which have a great influence on the calculation of fluid loads on offshore structures, especially in severe sea states.

In summary it appears justified to conclude that the linear random model provides a comprehensive and remarkably good description of the surface elevation and the wave kinematics, which is far more accurate than was generally believed thus far. On the whole it provides a quite adequate representation of a natural sea for engineering applications, which is preferable over and has a wider scope of applicability than the non-linear periodic wave models. The linear model has obvious limitations for accurate prediction of extreme events, but it may well be that theoretical and/or empirical modifications to the linear random theory have a better chance of achieving the required accuracy for such extrapolations than the non-linear periodic wave theories. Other areas for further work concern the description and influence of wave group effects and the wave kinematics around the still mean waterline.

10. Conclusions and recommendations

The following conclusions can be drawn.

General

a) The results presented in this paper support on the whole that the linear random wave model provides a valid and adequate theoretical description for the calculation of wave kinematics in uni-directional irregular waves in engineering applications.

b) Together with the possibility to include the influence of directional spreading in multi-directional seas (which influence is important for an accurate description of the velocity field in real conditions, as reported elsewhere) this suggests that the linear random model forms a solid basis for the calculation of wave kinematics in the offshore environment, although the linear model needs modifications in order to describe the most extreme events occurring during a sea state with a high degree of accuracy.

Specific

c) Surface elevation and horizontal velocities show an opposite crest/trough distortion for large amplitudes. Wave crests are higher than predicted by the theoretical distributions (Gauss distribution for instantaneous values and Rice distribution for amplitudes), whereas the wave troughs are less deep than follows from theory. Large horizontal crest velocities are smaller than can be derived from theory, while large horizontal trough velocities exceed the theoretical values. An explanation for this phenomenon can be found in the influence of second order low frequency energy on the crest and trough distributions of the horizontal velocities (see Section 9). The crest/trough distortion of the surface elevation and horizontal velocities generally increases with increasing steepness of the sea states (maintaining the same spectral peak period).

d) The statistical distributions of crests and troughs of the vertical velocity are rather symmetrical. The measured values in the tails of the distributions are somewhat higher than the theoretical values predict.

e) Gain and phase functions between the surface elevation and velocities in a vertical plane fit linear theory quite well. The agreement tends to be better for orbital velocities at the higher elevations and for wave spectra with longer periods.

f) Predictions of time series using a particular measured record as input give good results for elevations below the deepest wave trough. Around the still water level the linear calculations seem to overpredict both the horizontal and the vertical velocity.

g) Gain and phase functions between two similar quantities separated over a horizontal distance of 1/3 L_p do not differ much from linear theory. The prediction of time series over this distance also gives good results.

h) The calculations using (higher order) periodic wave theories to describe a specific individual wave show that the results for the example wave are worse than those which are obtained from the prediction of time series using the linear random wave model.

Further work and recommendations

As already mentioned before more data is available and will be analyzed in the future. This data concerns mainly the velocities around the still water level and measurements separated by a horizontal distance. This work will be included in the final report within the MaTS-programme. Specific recommendations for further work include:

i) The development of theoretical enhancements and/or empirical modifications to the linear random model in order to improve predictions of extreme events occurring during a sea state.

j) The description of wave kinematics in the free surface zone between the deepest trough and the highest crest.

k) The study of second order, low frequency energy associated with wave groups and its influence on the wave kinematics.

Acknowledgement

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List of main symbols

a	distance from the bottom of the flume
a_c	crest amplitude (maximum)
a_t	trough amplitude (minimum)
d	waterdepth
f	frequency
g	acceleration of gravity
$h(t)$	impulse response function
$\|H(f)\|$	gain function
H_s	significant wave height
H_z	zero-crossing wave height
L	wave length
L_p	wave length related to T_p
m_o	variance of the energy density spectrum
$S(f)$	energy density
t	time
T_p	spectral peak period
T_z	zero-crossing wave period
$u(t)$	horizontal velocity
$w(t)$	vertical velocity
$\gamma^2(f)$	squared coherence function
ε	spectral width parameter
$\eta(t)$	surface elevation
ξ	relative error
σ	standard deviation

Table 1 Test programme

WAVE SPECTRUM	T_p [s]	H_s [cm]	$4\sqrt{m_o}$ [cm]	ε_T	ε_m
31	0.93	2.9	3.0	0.46	0.48
32	0.93	3.5	3.5	0.36	0.39
33	0.98	6.8	6.8	0.32	0.42
41	1.28	4.9	5.1	0.45	0.50
42	1.14	8.8	9.0	0.46	0.53
43	1.28	12.4	12.4	0.38	0.51
51	2.05	5.7	5.9	0.53	0.59
52	2.05	11.5	12.0	0.53	0.62
53	1.86	17.5	18.0	0.52	0.65

Table 2 Statistical distribution of instantaneous values - relative error in % of the theoretical value (normal distribution) at 1% exceedance

WAVE SPECTRUM	SURFACE ELEVATION		HORIZONTAL VELOCITY						VERTICAL VELOCITY					
			a/d = 0.25		a/d = 0.50		a/d = 0.75		a/d = 0.25		a/d = 0.50		a/d = 0.75	
	POS	NEG	POS	NEG	POS	NEG	POS	NEG	POS	NEG	POS	NEG	POS	NEG
31	+ 0.1	+1.7	- 3.3	+ 2.9	-4.0	+ 3.1	-1.9	+ 3.0	+ 4.2	-3.8	+1.7	-1.5	+1.0	+0.7
32	+ 4.1	+4.1	- 6.1	+15.1	-1.8	+10.8	+0.8	+ 8.9	+12.1	-3.6	+6.8	+3.7	+4.8	+5.1
33	+12.5	+2.3	-12.6	+26.8	-6.7	+18.6	-2.5	+15.9	+ 8.6	+6.6	+6.9	+5.4	+5.5	+5.2
41	+ 0.9	+0.2	- 1.5	+ 7.9	-0.2	+ 6.7	-3.3	+ 5.3	+ 4.0	+1.9	+1.5	+2.4	-0.2	+2.0
42	+ 8.4	-3.2	- 4.6	+12.0	-3.6	+10.1	-3.4	+ 8.4	+ 3.0	+3.8	+2.7	+3.6	+2.0	+2.3
43	+11.5	-2.6	- 9.1	+19.6	-6.5	+16.7	-3.3	+11.7	+ 5.4	+5.2	+4.1	+4.2	+3.3	+4.4
51	+ 1.3	+0.9	- 2.7	+ 7.7	-2.3	+ 6.8	-1.5	+ 5.0	+ 2.9	+3.5	+3.1	+2.7	+3.3	+1.9
52	+ 5.5	-1.3	- 7.2	+10.0	-6.1	+ 9.1	-4.0	+ 7.9	+ 3.9	+3.6	+3.6	+3.3	+3.5	+2.1
53	+ 7.2	-5.2	-11.2	+11.0	-9.4	+10.1	-6.8	+ 7.5	+ 1.1	+2.7	+1.6	+3.5	+1.7	+3.7

Table 3 Statistical distribution of maxima and minima - relative error in % of the theoretical value (Rice distribution) at 1% exceedance

WAVE SPECTRUM	SURFACE ELEVATION		HORIZONTAL VELOCITY						VERTICAL VELOCITY					
			a/d = 0.25		a/d = 0.50		a/d = 0.75		a/d = 0.25		a/d = 0.50		a/d = 0.75	
	η_c	η_t	u_c	u_t	u_c	u_t	u_c	u_t	w_c	w_t	w_c	w_t	w_c	w_t
31	+ 3.6	+2.2	- 4.5	+ 3.6	- 4.3	+ 3.9	-1.7	+ 6.2	+ 3.0	-3.0	-0.5	-2.6	+6.4	+0.8
32	+ 8.0	+4.6	+ 0.9	+26.5	- 1.3	+14.0	+0.7	+11.3	+17.6	-1.4	+9.8	+6.3	+8.2	+5.6
33	+14.8	+2.6	-15.7	+31.7	-11.1	+22.8	-5.6	+17.4	+11.5	+5.8	+7.4	+4.1	+8.0	+6.8
41	+ 9.4	-1.1	+ 1.6	+12.2	- 0.2	+11.4	+0.9	+ 9.1	+ 7.7	+4.8	+6.1	+5.9	+6.5	+1.8
42	+17.0	-5.9	- 2.8	+18.6	- 2.9	+16.8	-4.0	+17.6	+ 6.9	+6.1	+6.2	+7.0	+7.0	+4.8
43	+19.0	-7.4	- 9.8	+18.8	- 6.7	+17.9	-3.6	+14.0	+ 5.7	+3.1	+6.0	+4.4	+5.9	+5.1
51	+ 9.3	0.0	+ 2.4	+ 9.0	+ 3.3	+ 8.5	+3.4	+ 5.7	+10.8	+1.4	+6.4	+1.0	+3.9	-0.3
52	+11.2	-2.6	- 5.1	+18.2	- 4.9	+13.9	-5.7	+12.4	+ 8.8	+2.4	+9.2	+1.3	+6.0	+2.7
53	+20.9	-9.3	-12.2	+10.1	- 9.9	+ 8.7	-6.7	+ 6.8	+ 0.4	+2.1	+3.4	+1.2	+2.1	+0.4

Table 4 Gain function (vertical plane) - average value of the magnitude of the relative errors in % of the theoretical values for frequencies for which $\gamma^2 \geqq 0.90$ (see equation 7.1)

WAVE SPECTRUM	SURFACE ELEVATION → HORIZONTAL VELOCITY			SURFACE ELEVATION → VERTICAL VELOCITY		
	a/d=0.25	a/d=0.50	a/d=0.75	a/d=0.25	a/d=0.50	a/d=0.75
31	5.4	7.3	4.7	8.9	2.6	3.2
32	5.4	8.0	8.0	13.4	3.5	5.0
33	6.6	9.8	6.0	10.0	8.3	4.0
41	4.7	4.6	3.3	1.5	2.5	1.6
42	5.4	4.5	2.0	3.1	2.3	2.0
43	8.5	8.6	3.1	4.8	3.3	2.9
51	5.9	5.3	2.8	5.2	2.0	2.4
52	3.4	3.5	2.3	3.5	2.4	1.8
53	4.2	3.3	4.4	4.5	1.3	3.1

Table 5 Phase function (vertical plane) - average value of the differences between measurements and theory in degrees for frequencies for which $\gamma^2 \geqq 0.90$

WAVE SPECTRUM	SURFACE ELEVATION → HORIZONTAL VELOCITY			SURFACE ELEVATION → VERTICAL VELOCITY		
	a/d=0.25	a/d=0.50	a/d=0.75	a/d=0.25	a/d=0.50	a/d=0.75
31	-0.6	-0.7	-2.7	+16.3	+8.4	+2.0
32	-0.3	-1.1	-1.5	+21.0	+9.0	+2.9
33	-0.9	-1.1	-1.8	+10.3	+4.0	+0.4
41	-0.5	-0.4	-1.9	+ 6.6	+3.5	-0.1
42	-0.7	-0.5	-0.6	+ 0.9	+2.6	-0.3
43	-0.9	-0.4	-0.2	+ 3.7	+1.5	-0.4
51	-0.1	-0.3	-0.9	+ 0.6	+2.9	+0.5
52	-0.5	-0.6	-0.3	+ 2.7	+1.9	+0.7
53	-0.3	-0.4	-0.3	- 0.2	+1.2	+0.2

Table 6 Gain function (horizontal separation 1/3 L_p) - average value of the magnitude of the relative errors in % of the theoretical values for frequencies for which $\gamma^2 \geqq 0.90$ (see equation 7.1)

WAVE SPECTRUM	SURFACE ELEVATION → SURFACE ELEVATION	HORIZONTAL VELOCITY → HORIZONTAL VELOCITY BOTH AT a/d = 0.75	VERTICAL VELOCITY → VERTICAL VELOCITY BOTH AT a/d = 0.75
31	4.7	2.2	2.9
32	4.9	2.5	3.3
33	4.3	2.5	3.0
41	5.0	4.7	1.5
42	3.6	4.1	2.0
43	4.6	3.4	2.4
51	4.0	4.9	1.3
52	3.9	5.4	1.7
53	4.0	5.6	1.4

Table 7 Phase function (horizontal separation 1/3 L_p) - average value of the differences between measurements and theory in degrees for frequencies for which $\gamma^2 \geq 0.90$

WAVE SPECTRUM	SURFACE ELEVATION → SURFACE ELEVATION	HORIZONTAL VELOCITY → HORIZONTAL VELOCITY BOTH AT a/d = 0.75	VERTICAL VELOCITY → VERTICAL VELOCITY BOTH AT a/d = 0.75
31	-1.9	-2.4	+2.2
32	-2.1	-2.9	+0.7
33	-3.7	-4.2	-0.3
41	-0.7	-0.9	+2.1
42	-1.9	-3.1	-0.5
43	-3.1	-2.6	-1.1
51	0.3	-0.7	-0.2
52	0.1	-0.3	-0.6
53	-1.1	-1.3	-1.6

Table 8 Statistical distributions determined from the measured and calculated time series (spectrum 43) - ratio of σ from calculations and measurements and relative errors in % of the calculated values at 1% exceedance

	ELEVATION	$\frac{\sigma_{calc}}{\sigma_{meas}}$	INSTANTANEOUS VALUES		MAXIMA AND MINIMA	
			POS	NEG	a_c	a_t
HORIZONTAL VELOCITY	a/d = 0.25	1.00	- 6.5	+ 7.8	- 4.4	+ 8.6
	a/d = 0.50	1.00	- 5.6	+ 6.8	- 4.3	+11.2
	a/d = 0.75	1.01	- 8.7	+ 6.2	- 9.6	+ 7.1
VERTICAL VELOCITY	a/d = 0.25	0.96	+ 5.2	+ 6.2	+ 8.0	+ 3.7
	a/d = 0.50	0.99	+ 1.7	+ 1.9	+ 5.1	+ 1.8
	a/d = 0.75	1.00	+ 2.2	+ 0.2	+ 2.0	+ 0.2
	HORIZONTAL SEPARATION					
SURFACE ELEVATION	1/3 L_p	1.02	+ 8.9	-12.4	+ 6.1	-15.6
HORIZONTAL VELOCITY AT a/d = 0.75	1/3 L_p	1.02	- 2.9	- 1.1	- 6.4	+ 4.5
VERTICAL VELOCITY AT a/d = 0.75	1/3 L_p	0.96	+ 0.7	+ 8.4	- 2.7	+ 9.8

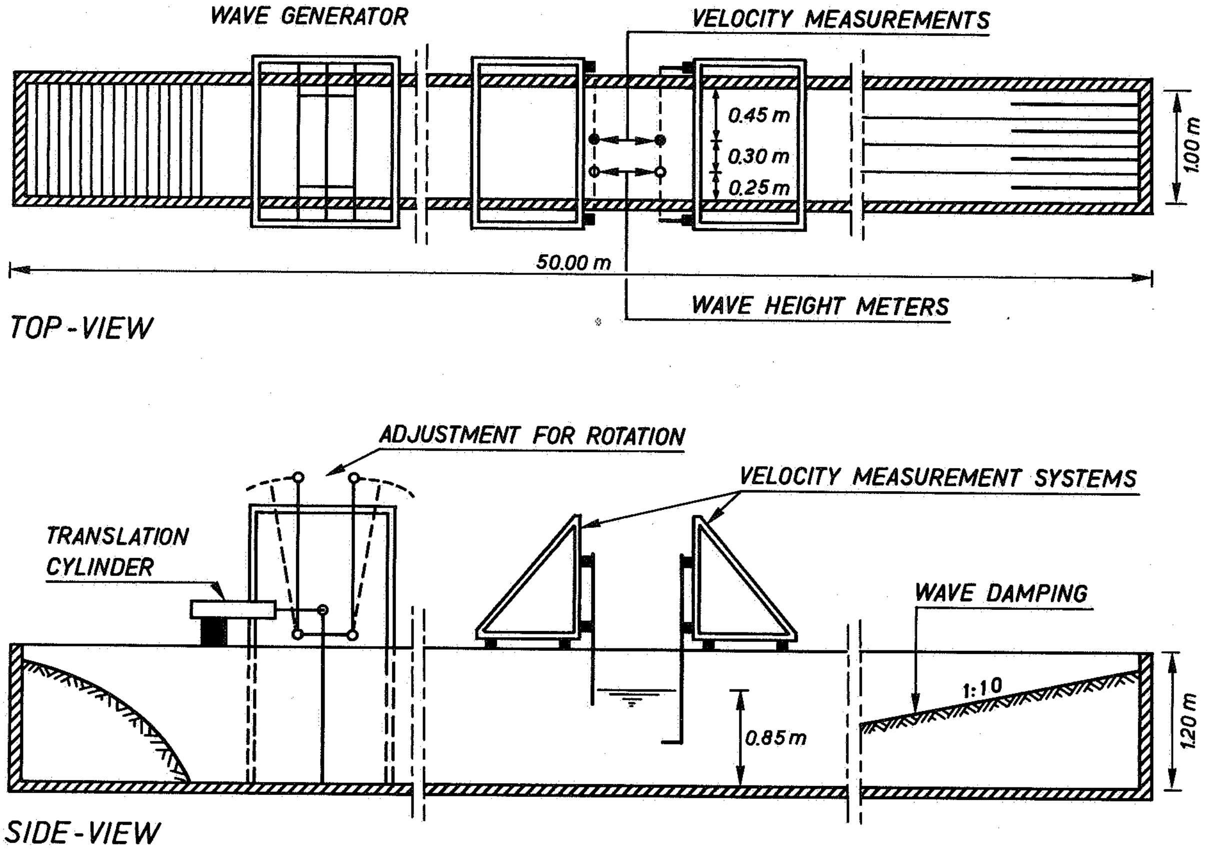

Fig. 1 Lay-out of the wave flume

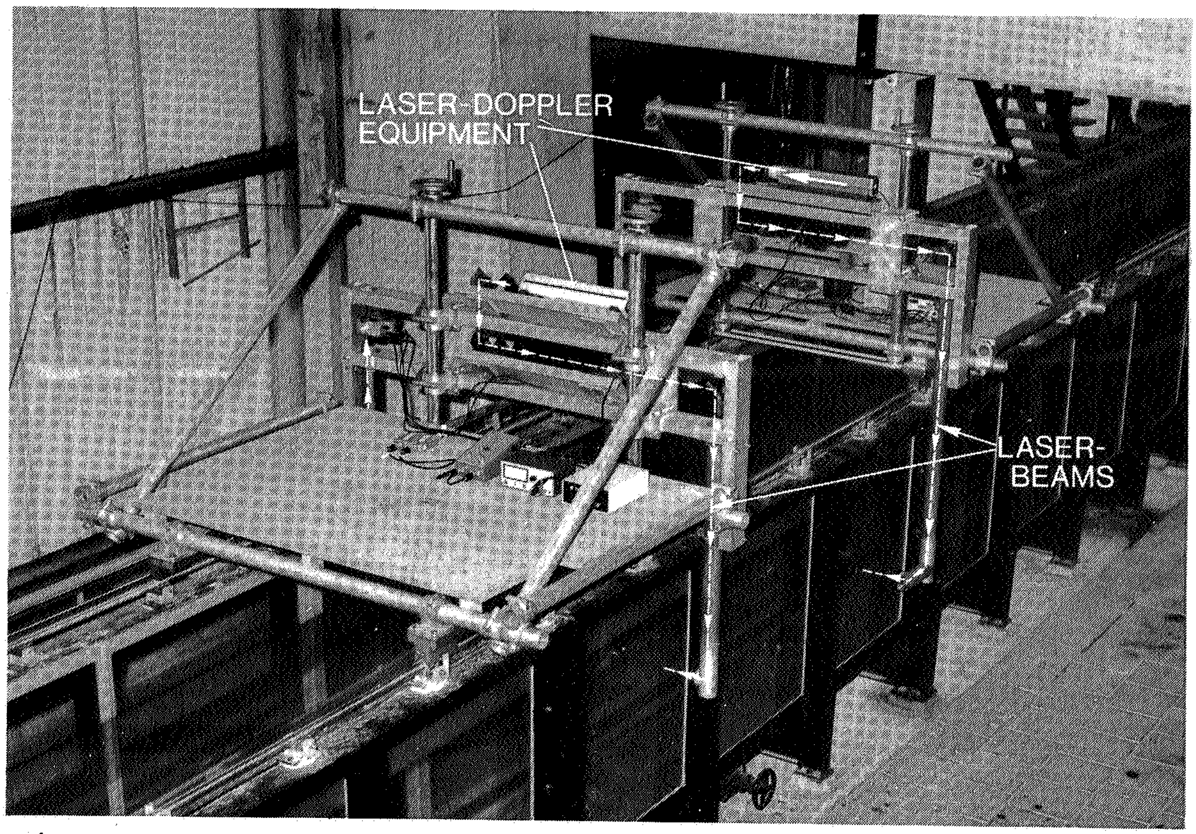

Fig. 2 Photo of the Laser-Doppler velocity measurement equipment

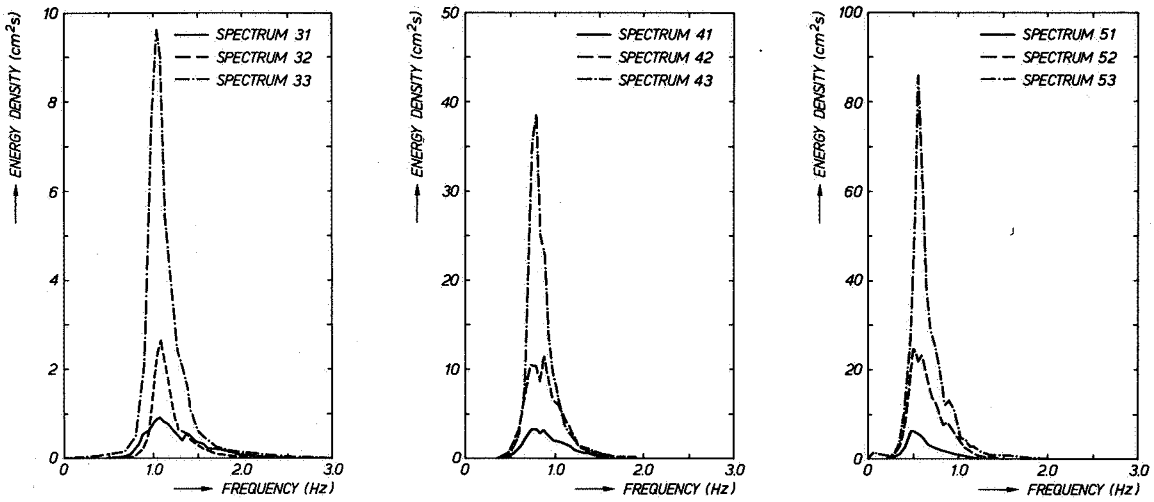

Fig. 3 Energy density spectra of the surface elevations (see Table 1 for spectral parameters)

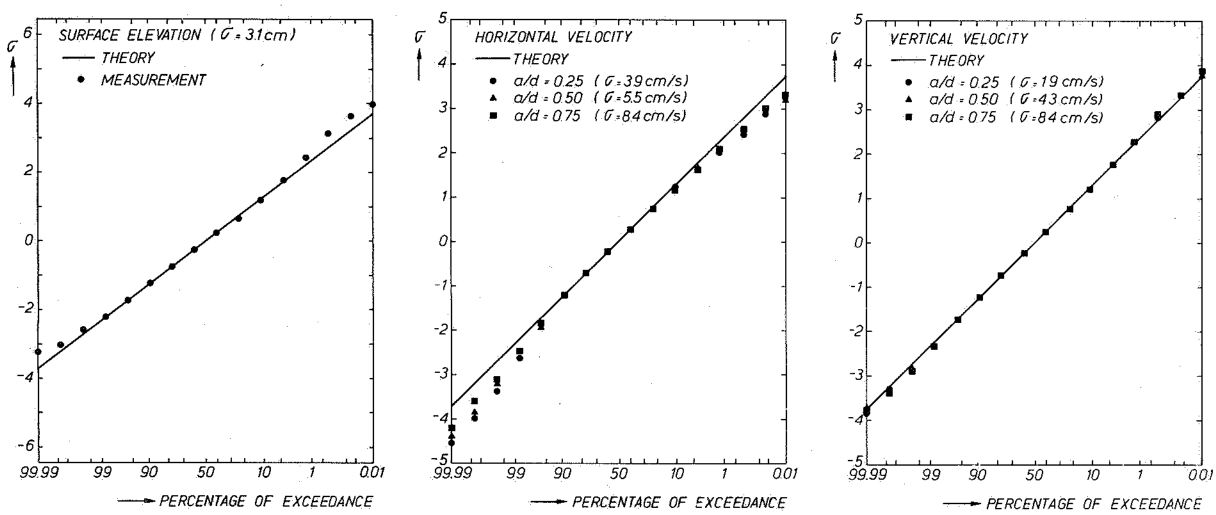

Fig. 4 Statistical distributions of instantaneous values (wave spectrum 43)

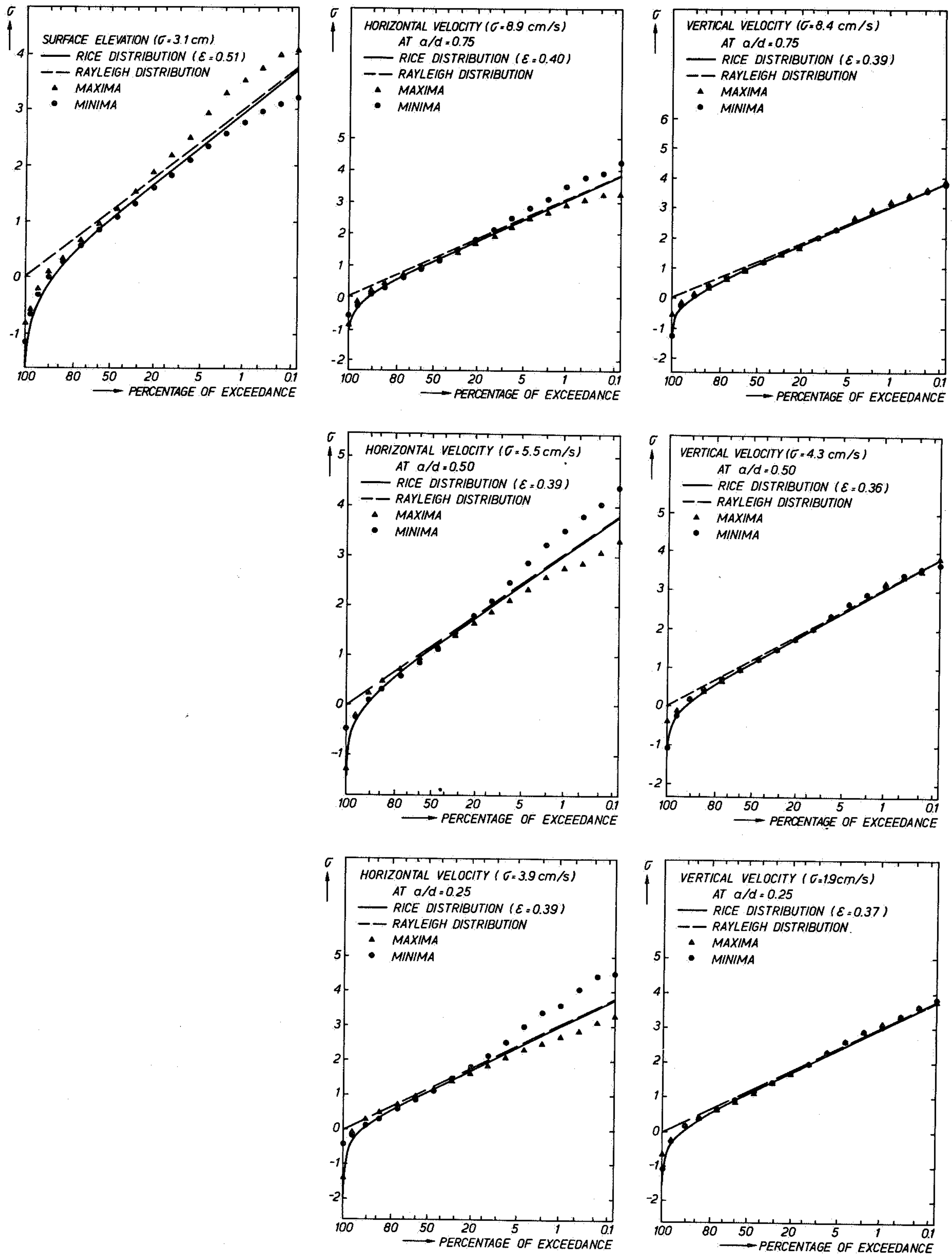

Fig. 5 Statistical distributions of maxima and minima (wave spectrum 43)

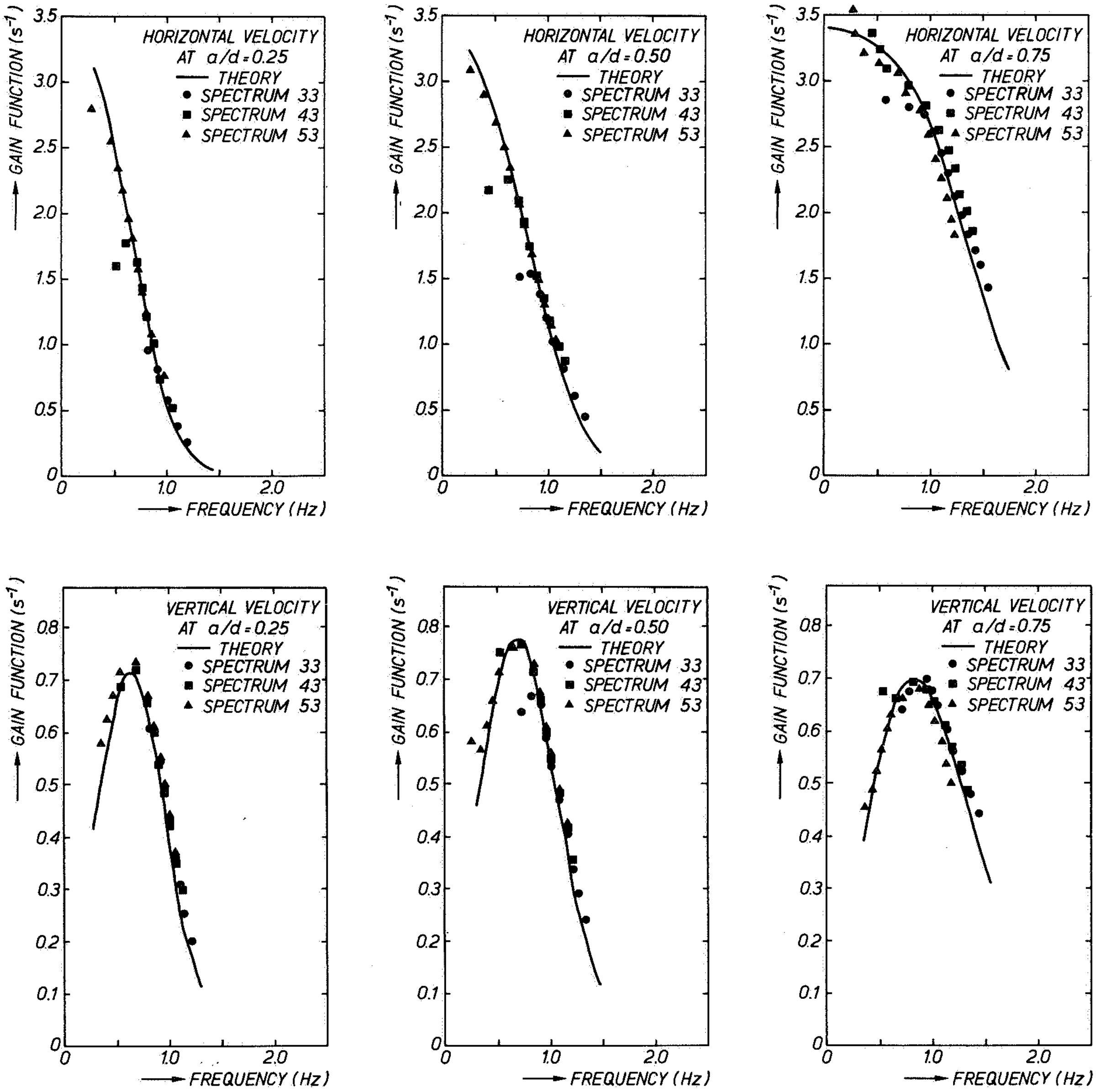

Fig. 6 Gain functions between surface elevation and orbital velocities in a vertical plane (wave spectra 33, 43 and 53)

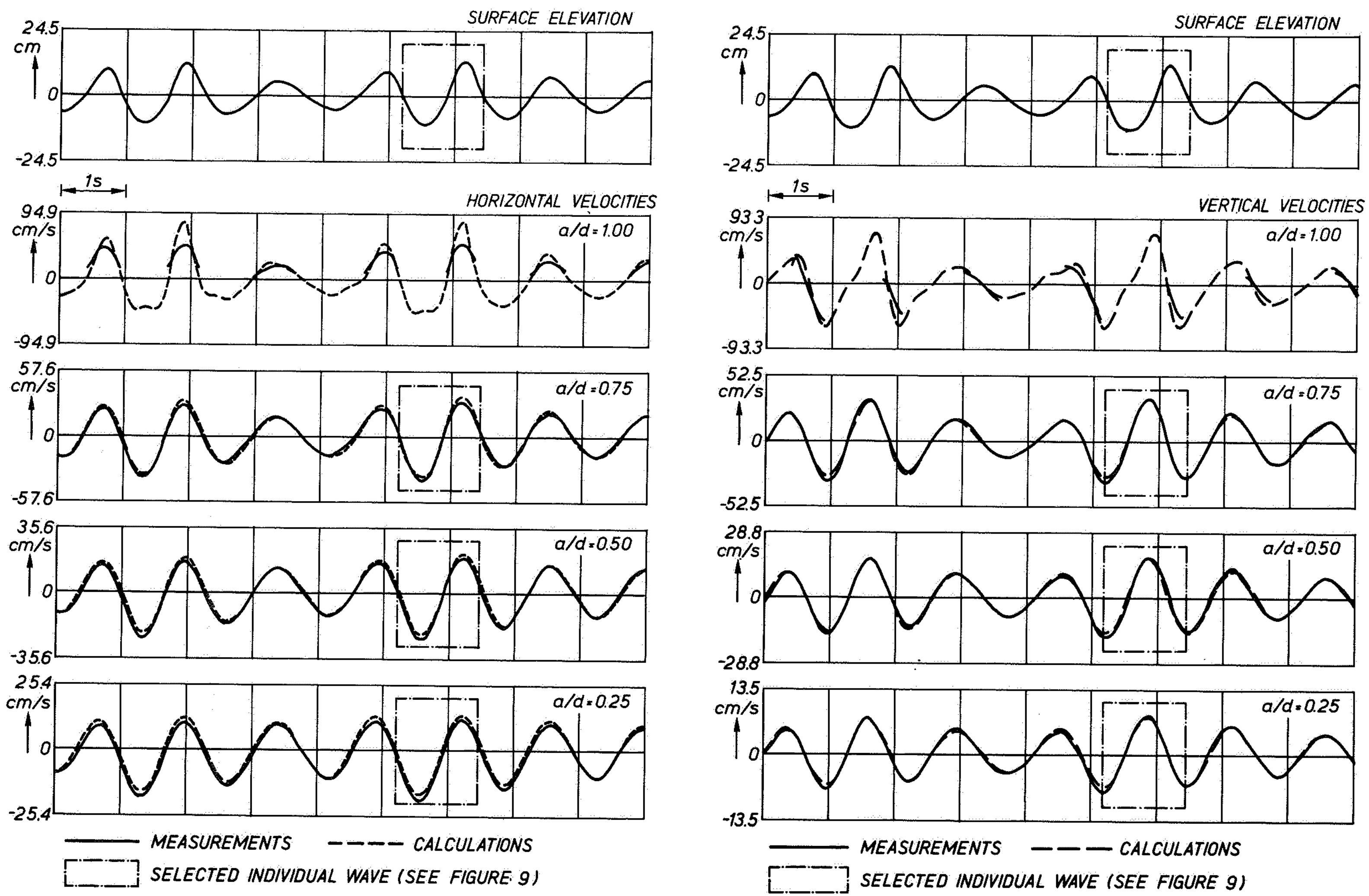

Fig. 7 Time series of surface elevation and orbital velocities - calculations in a vertical plane (wave spectrum 43)

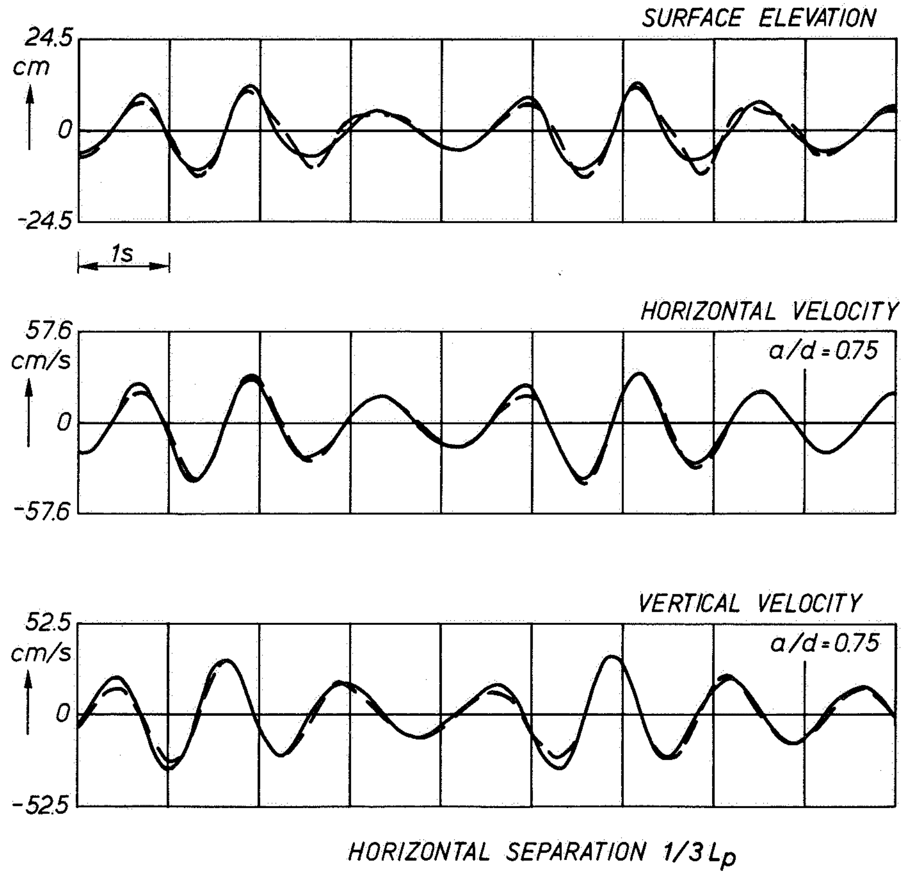

Fig. 8 Time series of surface elevation and orbital velocities - calculations over a horizontal distance (wave spectrum 43)

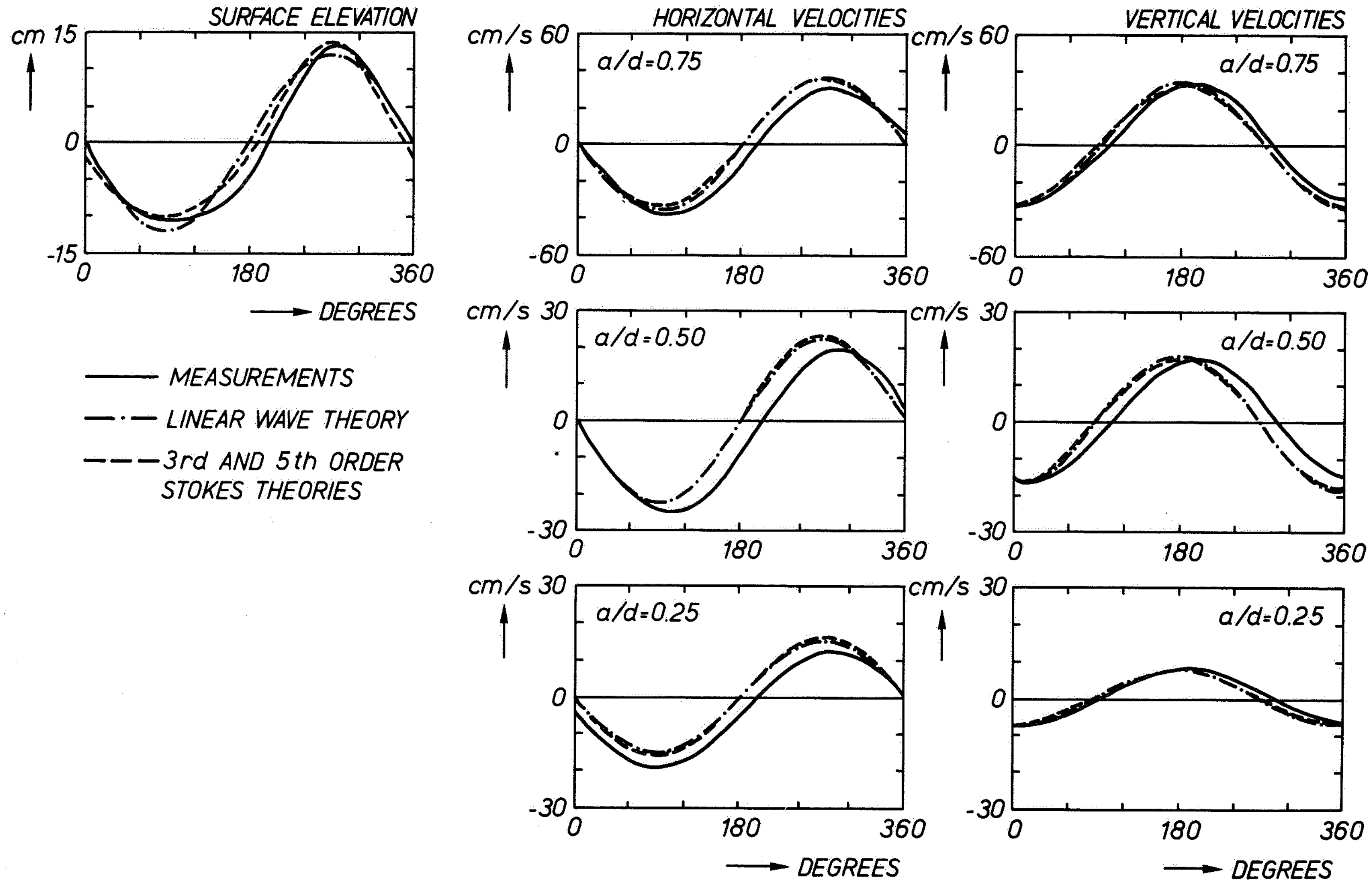

Fig. 9 Comparison of an individual wave and orbital velocities with periodic wave theories